Predicting Changes in Disc Golf Scoring Distributions from Changes in Hole Design
March 23, 2014
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Abstract

Disc golf course designers manipulate the characteristics of a disc golf hole in order to achieve better scoring distributions. This paper presents a method by which a designer can predict how the scoring distribution for a targeted skill level will be affected by incremental changes to the difficulty of a hole.

The method is to

1) use scores from a variety of players of different ratings to compute scoring distributions for a range of ratings,

2) use the scoring distributions for ratings that are incrementally higher or lower than the targeted skill level as an approximation of the scoring distributions that would result from making the hole easier or harder.

Quantifying the Scoring Distribution of a Hole

If the group of players being considered is the same as the group of players for which information is available, the calculation of the scoring distribution is straightforward.

For example, on Hole #8 at a recent tournament[1] over two rounds, the 64 players recorded 2x2s, 52x3s, 52x4s, 12x5s, and 4x6s. This gives a scoring distribution of

Table 1

All Players

Score	Frequency
2	1.6%
3	42.6%
4	42.6%
5	9.8%
6	3.3%

However, these players were at all different skill levels. To calculate the scoring distribution of a hypothetical group of players who are all at the same skill level takes a little more work.

One approach which has been used is to select a group of players whose ratings are near to - and average out to - the skill level to be studied. Say the course is being optimized for players rated 950. In this tournament, there were 34 players whose ratings were

[1] Everyday Fall Open, held at The Valley Disc Golf Course September 28, 2013.

between 900 and 1000. The average rating of these players was 946. So, the 4 players at the low end would be dropped to bring the average closer to 950.

This is the same as setting a center and a range (955 +or– 44) and assigning each score within that range a weighting of 1.
Using this method, the scoring distribution for 950-rated players on Hole 8 would be:

Table 2

950 Average	
Score	Frequency
2	0.0%
3	33.9%
4	53.2%
5	12.9%
6	0.0%

This works OK as a point estimate, if the frequency of each particular score is linear as a function of rating. However, the frequency of any particular score is not linear as a function of rating. The frequency of a particular score will rise and fall again as rating varies. This tendency to peak can be captured with a more refined approach.

If the weightings are

exp(- ((Center - Rating) / Range) ^2)

the peaks can be reproduced. The weightings never go to zero, so scores from all players are used, capturing more information about infrequent scores. Using these weights also eliminates sharp changes in estimated frequencies.

In this case, with a Range of 35, and a center of 954.3, the estimated scoring distribution for 950-rated players would be:

Table 3

950 Weighted Average	
Score	Frequency
2	0.0%
3	36.3%
4	52.0%
5	11.7%
6	0.0%

Snapshot Results

Using the method described in the previous section and the hole-by-hole tournament results, I calculated the scoring distributions and some related statistics for each hole for 950-rated players:

Table 4

Hole	2	3	4	5	6+	Average Score	Highest Frequency	Scoring Spread Width
1	35.7%	56.9%	5.0%	2.4%	0.0%	2.74	56.9%	2.53
2	0.0%	18.8%	59.8%	20.8%	0.6%	4.03	59.8%	2.66
3	39.6%	53.0%	7.3%	0.2%	0.0%	2.68	53.0%	2.47
4	32.6%	59.5%	7.1%	0.8%	0.0%	2.76	59.5%	2.46
5	6.6%	52.3%	32.5%	5.0%	3.7%	3.48	52.3%	3.25
6	35.0%	53.9%	11.1%	0.0%	0.0%	2.76	53.9%	2.57
7	19.9%	61.2%	14.2%	4.7%	0.0%	3.04	61.2%	2.84
8	0.0%	36.3%	52.0%	11.7%	0.0%	3.75	52.0%	2.61
9	36.1%	50.1%	12.8%	1.0%	0.0%	2.79	50.1%	2.78
10	32.2%	56.4%	11.3%	0.0%	0.0%	2.79	56.4%	2.55
11	0.0%	30.9%	43.0%	22.7%	3.4%	3.99	43.0%	3.27
12	27.9%	58.4%	13.6%	0.2%	0.0%	2.86	58.4%	2.59
13	1.1%	53.3%	38.3%	7.2%	0.0%	3.52	53.3%	2.57
14	0.0%	23.1%	47.3%	25.3%	4.3%	4.12	47.3%	3.33
15	2.4%	47.4%	37.5%	10.8%	1.9%	3.62	47.4%	3.09
16	4.7%	59.4%	31.9%	2.7%	1.3%	3.36	59.4%	2.64
17	14.6%	66.9%	18.0%	0.5%	0.0%	3.05	66.9%	2.43
18	2.6%	50.9%	40.1%	4.8%	1.6%	3.52	50.9%	2.76

Much has already been written about using this information. We could stop here, having found a better way to calculate the scoring distribution for a particular rating of interest.

However, we can do more. What this snapshot cannot do is to predict how changes to the holes would affect the scoring spread.

For example, say we arbitrarily made Hole 8 easier by making it 35 feet shorter. We know[2] that its average score would drop from 3.75 to 3.64. What we don't know is whether that would that happen by all the 5s becoming 4s. Or if would it happen by 12% of all players shaving a throw off their score. Or some other pattern.

[2] From the Hole Forecaster, or any linear fit of score to length

The Leap of Faith

There are two ways to reduce the average score of a hole:

1. Make the hole easier.
2. Get better players to play the hole.

The question is: Are these analogous? Will the resulting scoring distribution be the same for both? Or at least reasonably close?

Without proof, I choose to believe that the result of making a hole easier is much like the result of having better players play a hole – at least within the limited range of what could be called tweaking the hole.

The rest of this paper is based on the premise that looking at the scoring distributions of better players on the same hole is like looking at the scoring distribution of an easier version of the same hole. And the flip side, that less skilled ≈ more difficult.

Back to Hole 8

Using the Hole Forecaster we can calculate that if the average score for 950-rated players on this 535-foot hole is 3.75, then the average score for 950-rated players if the hole were only 500 feet long would be about 3.64.

So, if we want to know the scoring distribution of Hole 8 if were only 500 feet instead of 535, we can get an idea by looking at the scoring distribution of Hole 8 for players with a weighted average rating high enough to produce an average score of 3.64.

By trial and error, we can find that a rating of 975 produces an average score of 3.64. The scoring distribution of players at that rating is compared to the scoring distribution of players at the 950 rating of interest.

Table 5

Rating	2	3	4	5	6+	Average Score	Highest Frequency	Scoring Spread Width
950	0%	35%	53%	12%	0%	3.75	52%	2.61
975	0%	40%	57%	3%	0%	3.66	57%	2.21

So, 975-rated players would get a lower score by getting fewer 5's and more 3's compared to 950-rated players. However, they would not score any 2's.

I think it is not too much of a stretch to think that for 950-rated players, moving the tee 35 feet forward would have the effect of turning most of the 5's into 4's and some of the 4's into 3's. Or, that a 500 foot hole is still too long to expect any 2s from 950-rated players.

We can thus surmise that shortening the hole would reduce the average score, make the most frequent score even more frequent, and also narrow the scoring spread.

Optimizing a Hole

Let's look at maximizing the Scoring Spread Width on Hole 8 for 950-rated players.[3]

We have seen that shortening the hole would narrow the Scoring Spread Width. What about lengthening it? How much longer is ideal?

We can compute the scoring distribution for all ratings for a hole. By looking at the resulting function of Scoring Spread Width vs. rating, we can see the ratings where the scoring spread is widest.

We can also see the average scores for the ratings where the scoring spread is widest. By translating the average scores into the equivalent lengths, we can generate a best guess for the optimal length of a hole.

On page 16 are charts for Hole 8.

The upper chart shows the frequency of each score as a function of rating. Each vertical slice is a scoring distribution for a particular rating. See that 52% of 950-rated players score a 4, for example.

The dotted gray line on the bottom chart is also familiar. This shows the average score as a function of rating. As expected, higher rated players get a lower average score.

The black line on the lower chart shows the width of the scoring spread as a function of rating.

For Hole 8, the Scoring Spread Width gets larger when the rating is lower than 950. This implies that the scoring spread for 950-rated players would be wider if the hole were harder. Maybe all the way to the point where the average score would be 4.28, or about a 675 foot long hole.

I suspect that the similarity between lower rated players and harder holes might break down before that point. Do 860-rated players on a 535 foot hole really get all the same scores as 950-rated players on a 675-foot hole? Maybe, maybe not.

However, I think we can be fairly sure that a modest increase in length would produce a wider scoring spread for 950-rated players. Because of this analysis, 2014 Am Worlds will likely use the longer target location.

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[3] Other goals could be sought instead: Targeting a certain average score, Minimizing the frequency of the most frequent score, Or trying to get 20% of the players to score a 5.

Hole 6: An Example of Widening a Scoring Spread by Making a Hole Easier

See the charts on page 14. The widest scoring spread is where the number of 2s is equal to the number of 3s. (Because the frequency of 4s doesn't change much over a wide range of difficulty.) So, because 950-rated players are currently getting more 3s than 2s, shortening the hole would widen the scoring spread.

Optimizing All the Holes

I looked for the ratings which has the widest scoring spread, took the average score of that ratings, then translated that average score into an equivalent length of hole[4]. The result is the length of hole for which the Scoring Spread should be widest.

Table 6

	Before			After			Change
Hole	Feet	Average Score	Scoring Spread Width	Feet	Average Score	Scoring Spread Width	Feet
1	375	2.74	2.53	375	2.74	2.53	0
1	375	2.74	2.53	666	3.45	2.71	291
2	500	4.03	2.66	599	4.46	3.38	99
3	170	2.68	2.47	203	2.84	2.81	33
4	240	2.76	2.46	256	2.82	2.60	16
5	315	3.48	3.25	383	3.84	3.78	68
6	265	2.76	2.57	244	2.69	2.65	-21
7	355	3.04	2.84	355	3.04	2.84	0
8	535	3.75	2.61	681	4.28	3.07	146
9	295	2.79	2.78	378	3.06	2.89	83
9	295	2.79	2.78	516	3.50	3.78	221
10	325	2.79	2.55	331	2.81	2.67	6
10	325	2.79	2.55	474	3.23	3.06	149
11	625	3.99	3.27	763	4.46	3.83	138
12	240	2.86	2.59	298	3.11	3.05	58
12	240	2.86	2.59	469	3.84	3.76	229
13	320	3.52	2.57	379	3.83	3.28	59
14	465	4.12	3.33	574	4.66	3.99	109
15	395	3.62	3.09	366	3.49	3.12	-29
15	395	3.62	3.09	428	3.77	3.10	33
16	430	3.36	2.64	761	4.54	4.41	331
17	340	3.05	2.43	411	3.30	2.87	71
18	465	3.52	2.76	528	3.75	2.88	63

A few observations. For the most part, making holes longer widens the Scoring Spread. This is to be expected. Longer, more difficult holes simply have more scores to work with (4s, 5s, 6,s etc.).

[4] Average Score minus 33/18 is proportionate to length of hole.

However, for all these holes except Hole 18, there was an optimal point somewhere short of infinitely more difficult where the scoring spread was widest. Even for Hole 18, there was a local maximum somewhere short of ridiculously long.

I would be cautious about large changes in difficulty. The very nature of the hole might change.

However, if the optimal point is in the direction of make-the-hole-harder, small increases in the difficulty of the hole can make the scoring spread get wider. On Hole 16, 60% of 950-rated players get a 3. Pushing the target farther back and up into the woods would certainly result in more 4s and fewer 3s. More balance among scores results in a wider scoring spread.

A couple of holes are already at an optimal point. Holes 1 and 7 would have a narrower scoring spread for 950-rated players if they were made either a little easier or a little harder. (Hole 1 might have an even wider scoring spread if it was made a whole lot harder, but small changes wouldn't help.)

The change in average score is better indicator of how much a hole needs to be changed than the change in feet. If a hole has a challenge other than pure length, then toughening up that challenge will probably give better results than adding length.

<u>Prioritizing</u>

Table 6 on page 6 shows the changes which would achieve the largest widening of scoring spreads. We see that for most holes we could probably widen the scoring spread by tweaking the holes. But is it worth it?

None of these holes has a narrow scoring spread. None really need to be adjusted.

Also, if we adjusted every hole for the maximum possible scoring spreads, we would end up mostly lengthening the holes. The course would be lengthened from 6,950 feet to 9,067 feet. The average score for 950-rated players would go up from 61.7 to 69.4. That might make the course too difficult to be fun.

On the other hand, if such an increase in difficulty were desired, we know where to do it.

If we want to adjust the course without a lot of changes, what we might like to do is focus on the holes that would produce a larger increase in scoring spread for a smaller change in average score. Fortunately, that ratio is easily computed.

Optimizing holes 3, 4, 6, 15, and 18 would add just 0.25 throws to the course score, but increase the scoring spread of those holes by a total of 0.71.

Hole 10 – What's the Deal With That?

Hole 10 (see page 17) has the strange property that players rated 970 have a slightly higher average score than players rated 930. This is a result of "better" players scoring 4s more often (the 2s and 3s are acting as they should).

This is a 325 foot hyzer throw that plays downhill for an effective length of 270. Perhaps the higher rated players overthrow the hole. Or, perhaps it is just a statistical fluke.

Because the average score does not go down as rating goes up, the analogy of better players ≈ easier hole breaks down.

However, we can still see that the scoring spread could be widened by increasing the number of 2s relative to 3s, implying that the shorter hole would have a wider scoring spread. Perhaps a shorter hole would trick some of the 950-rated players into acting like the 980-rated players do on the existing hole so the 950-rated players would get more 4s as well.

In any event, we would cross this hole off the list of holes to be made longer.

Graphs for All Holes

The following pages give a peek under the hood of all the holes.

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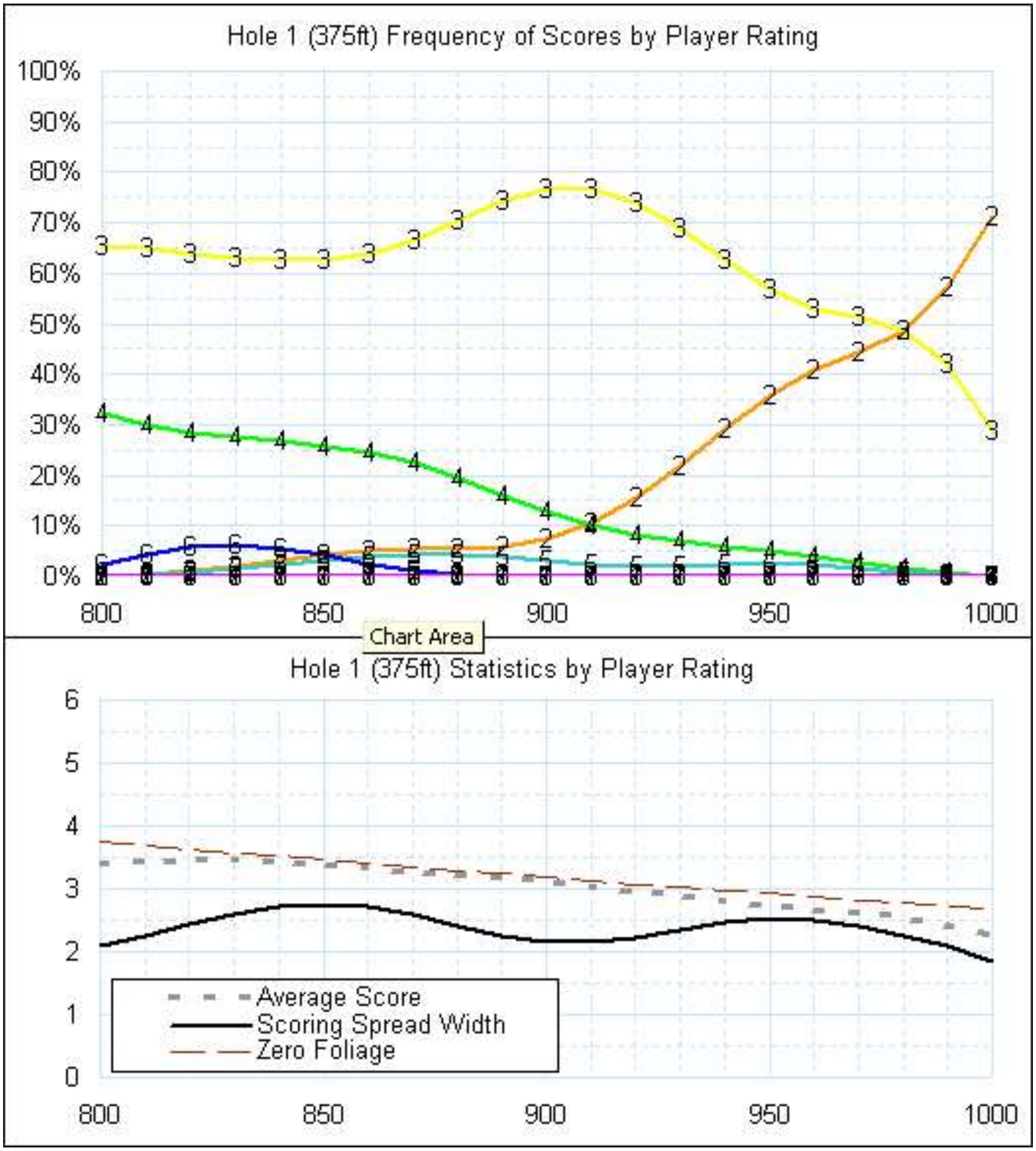
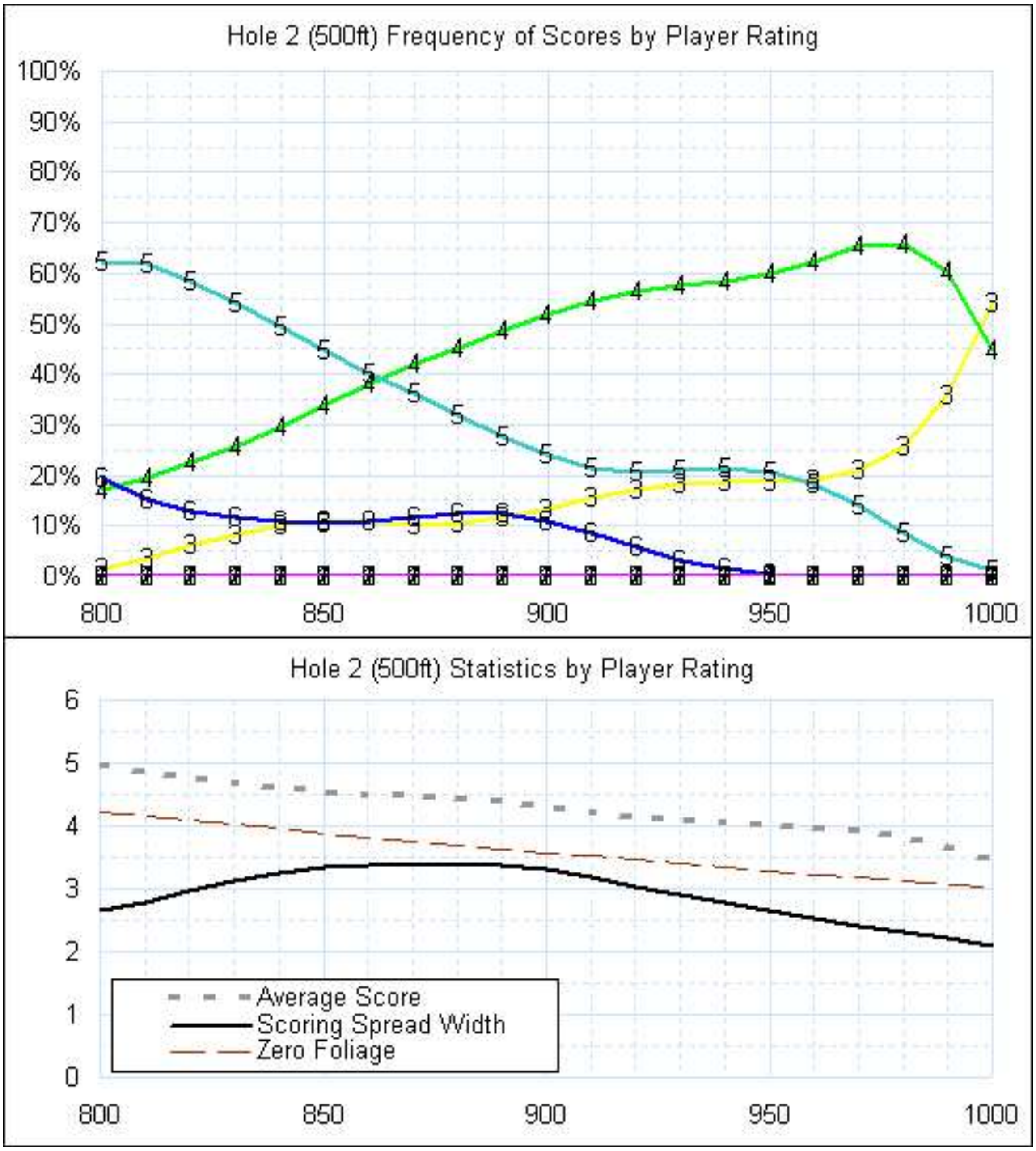


Hole 1 (375ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Chart Area
Hole 1 (375ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

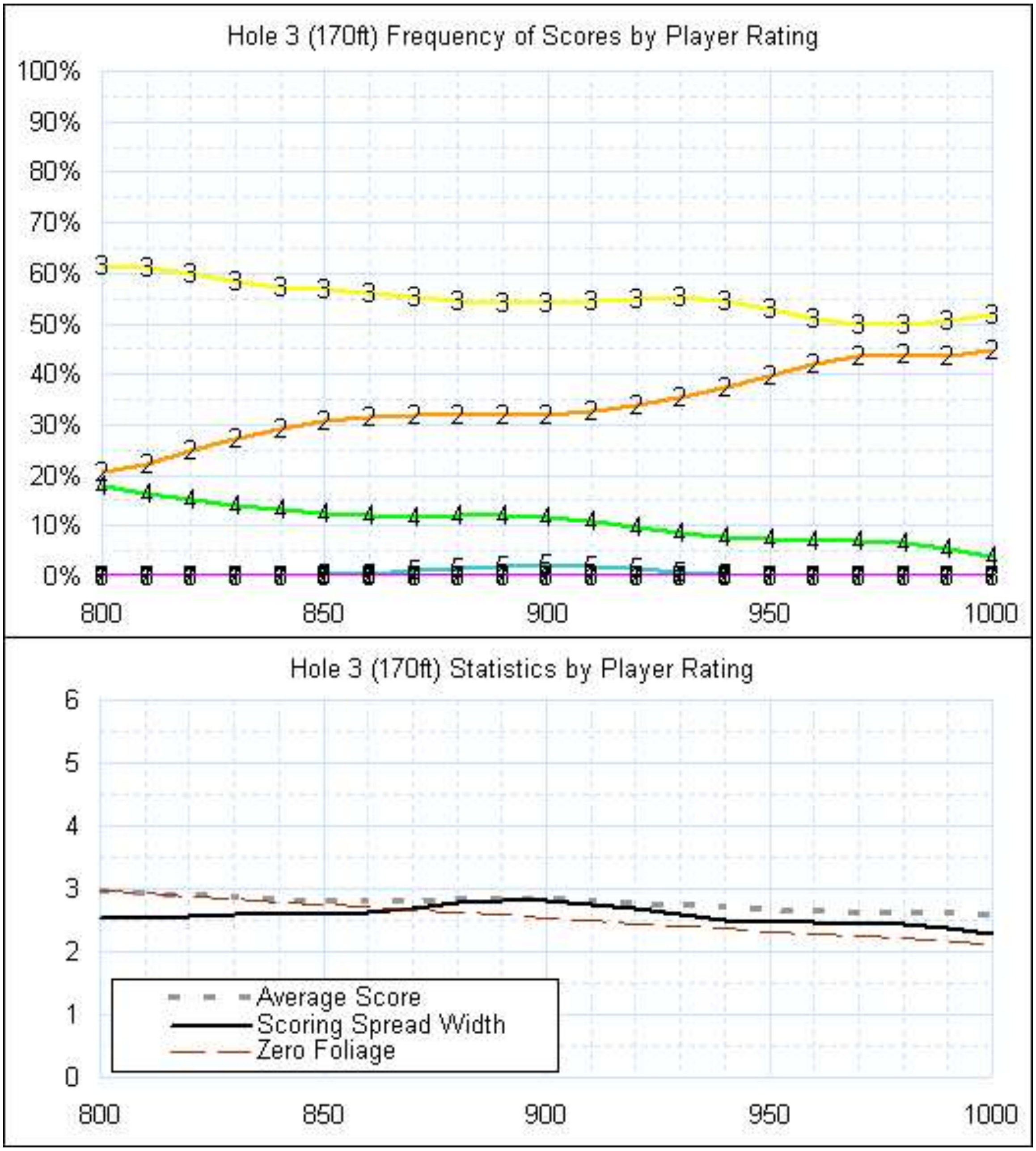
Hole 1

-


Hole 2 (500ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 2 (500ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

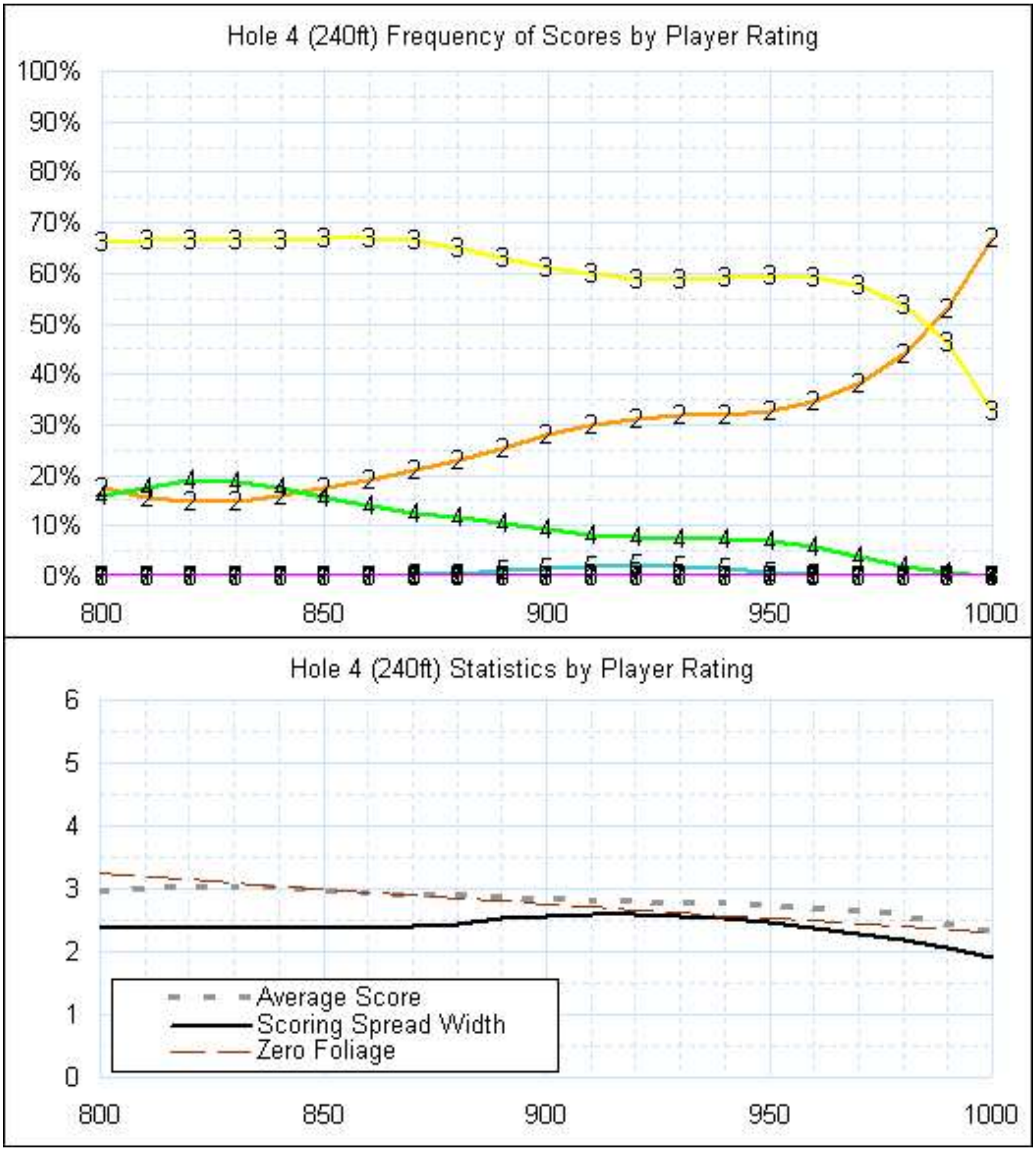
Hole 2

-


Hole 3 (170ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 3 (170ft) Statistics by Player Rating
6
5
4
3
2
1
0
800
850
900
950
1000
Average Score
Scoring Spread Width
Zero Foliage

Hole 3

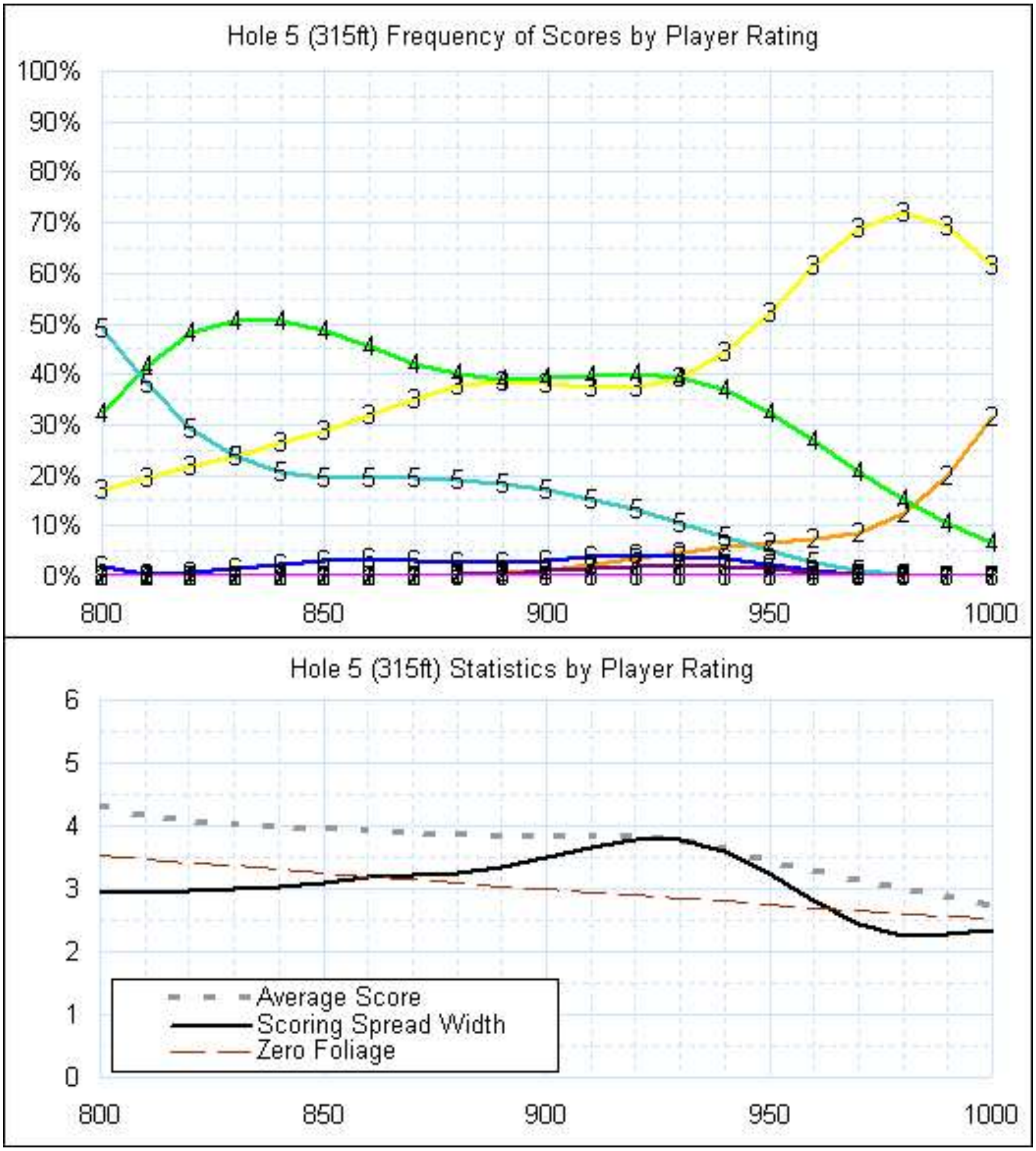
-



Hole 4 (240ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 4 (240ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

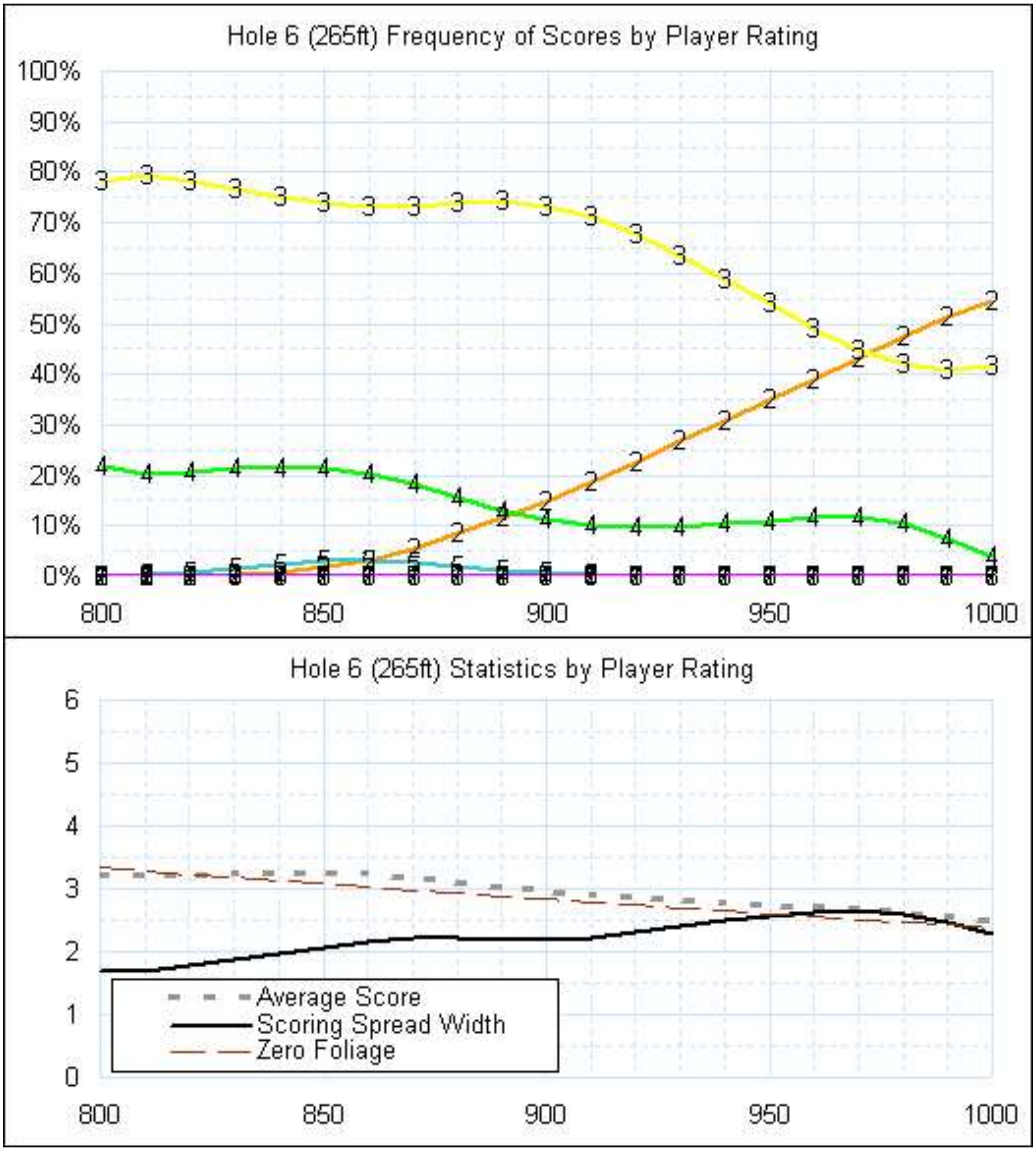
Hole 4

-

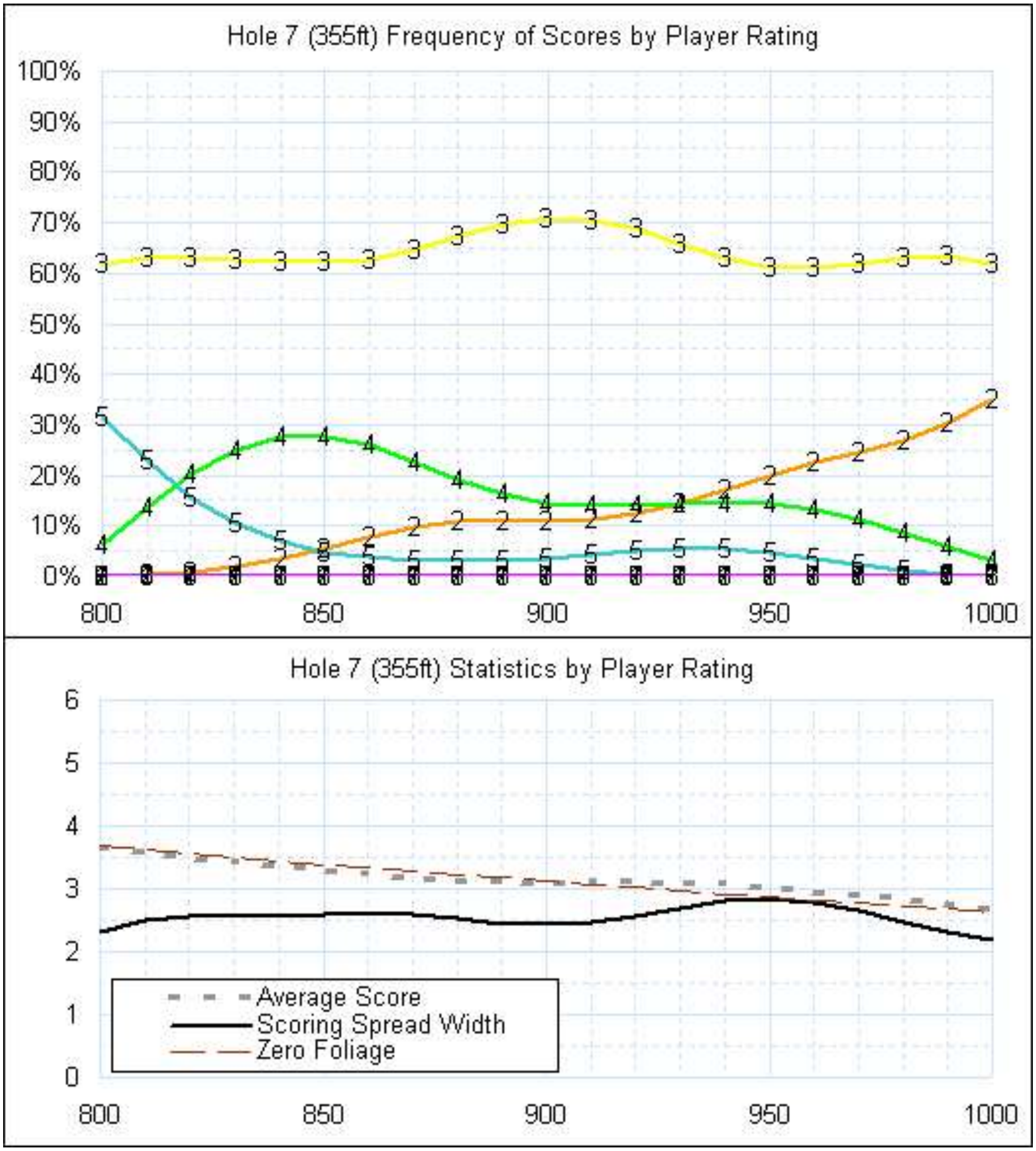

Hole 5 (315ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 5 (315ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

Hole 5

-


Hole 6 (265ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 6 (265ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000
Hole 6

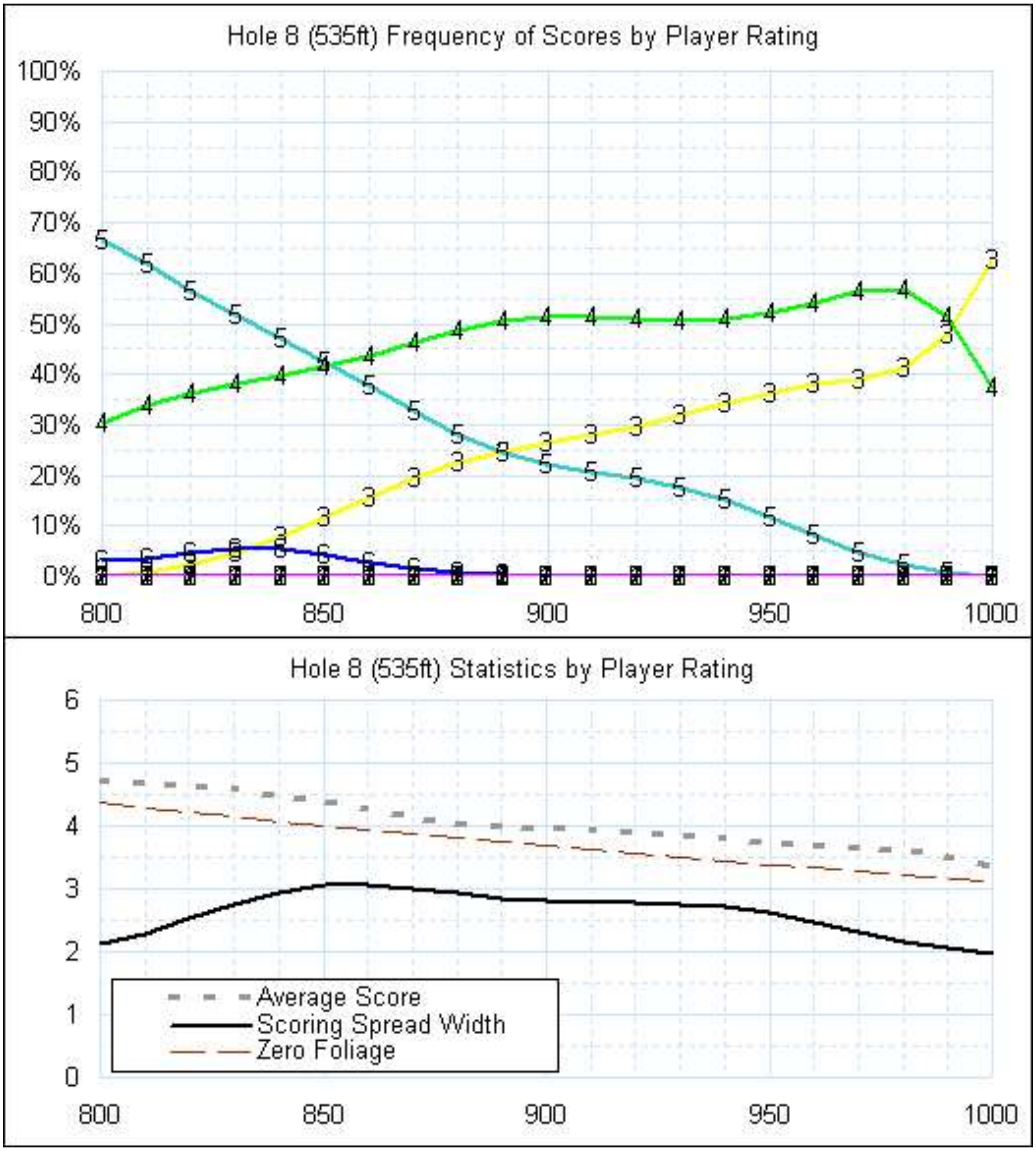
-



Hole 7 (355ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 7 (355ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

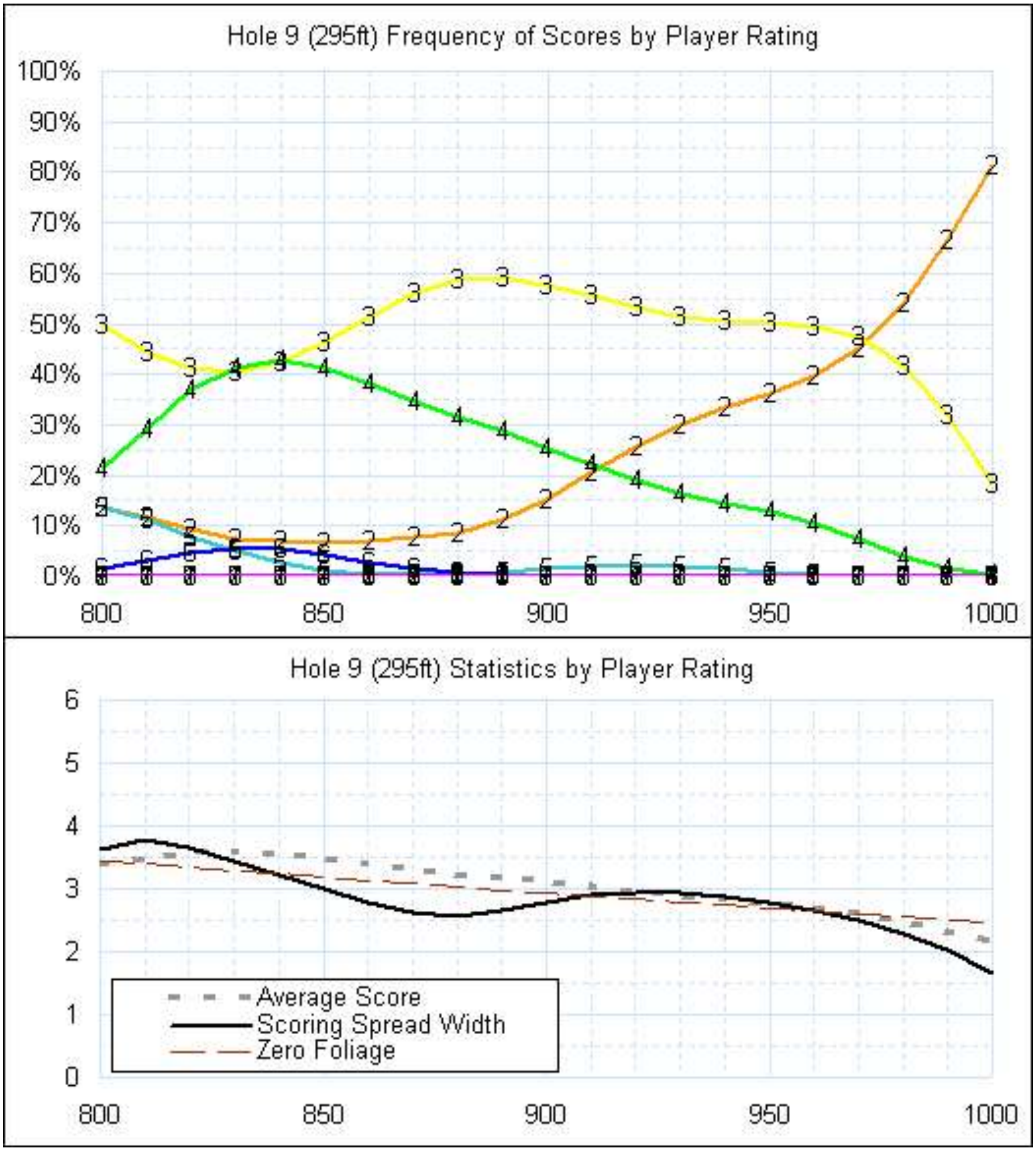
Hole 7

-


Hole 8 (535ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 8 (535ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 8

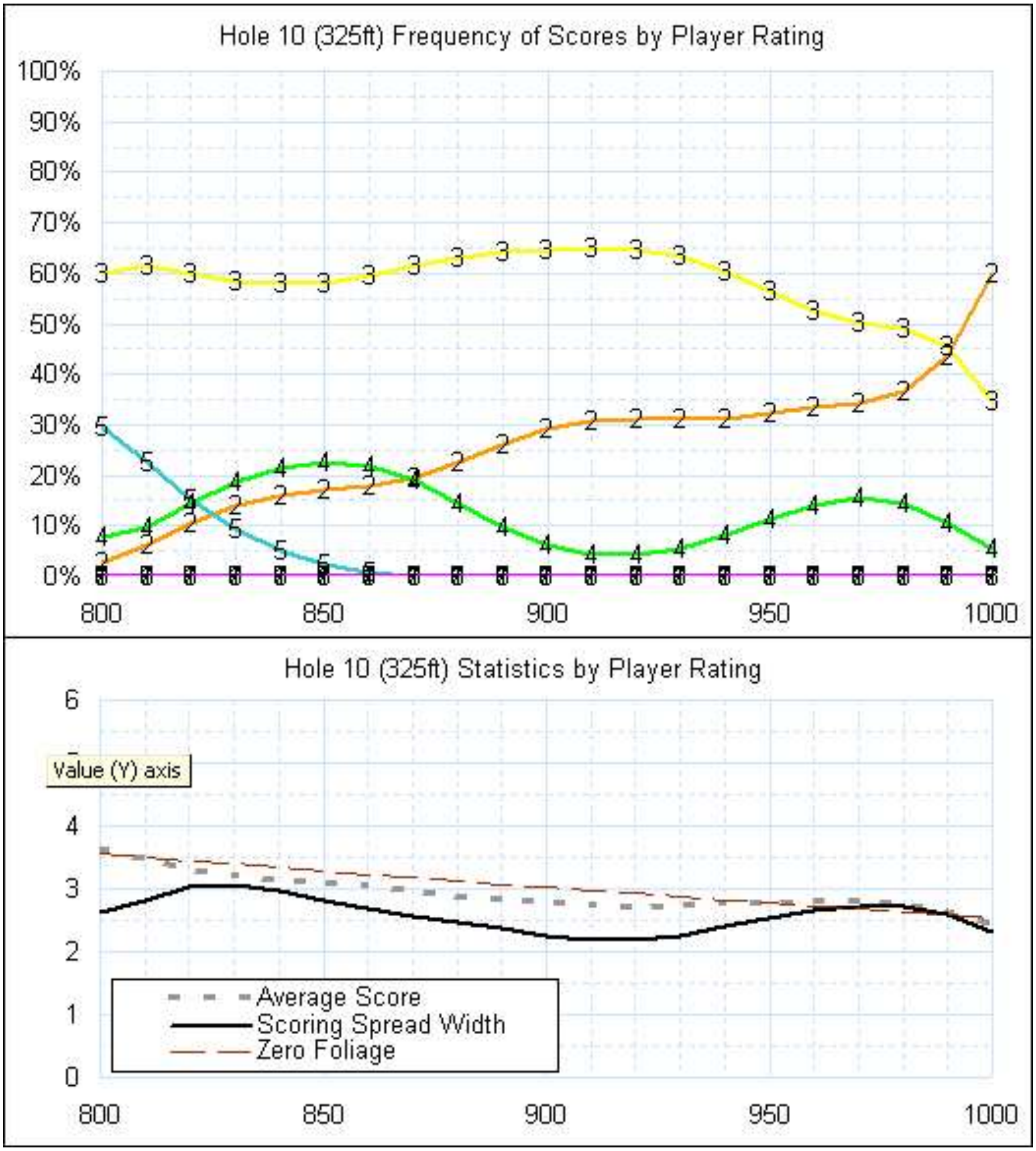
-



Hole 9 (295ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 9 (295ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 9

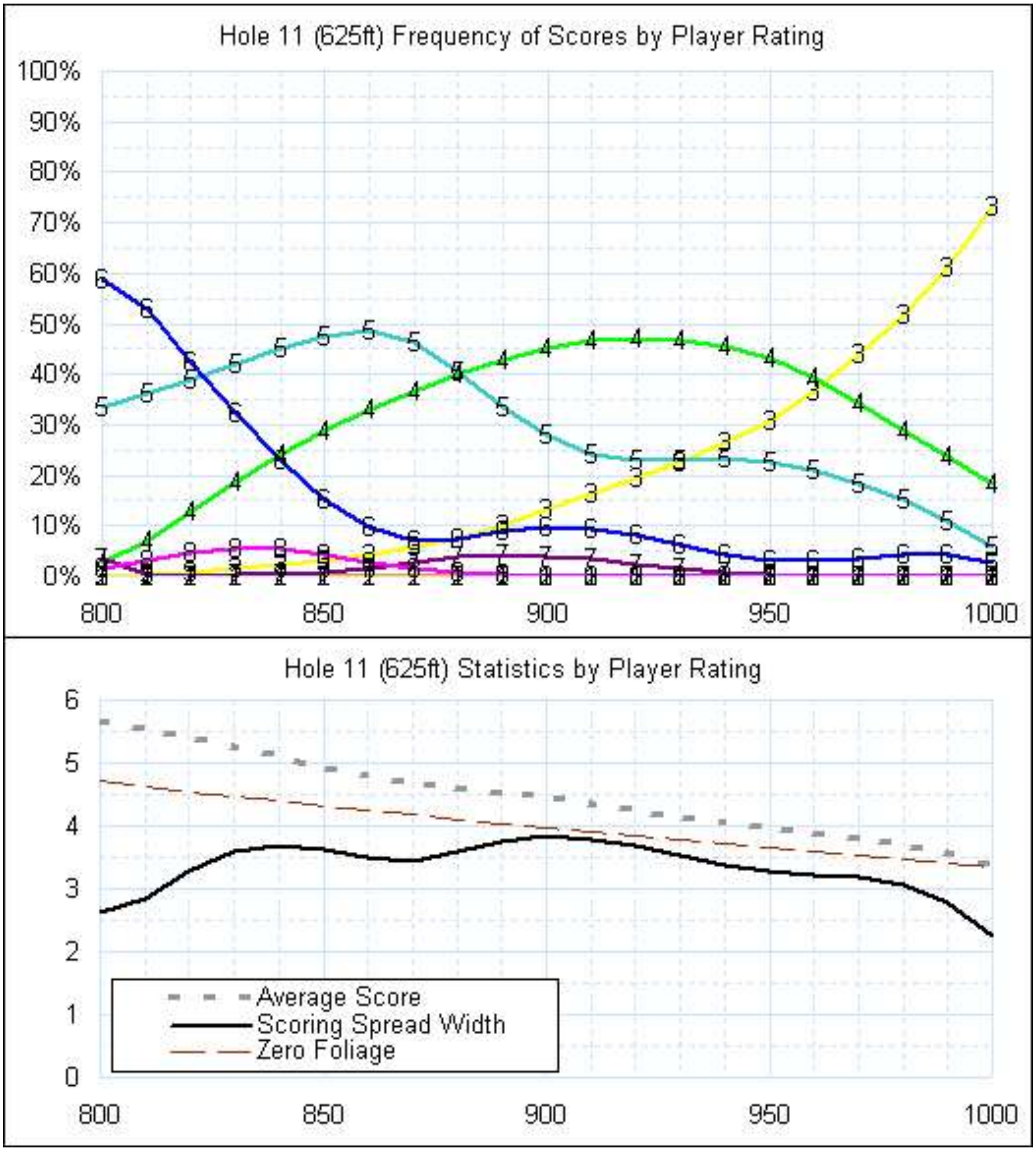
-



Hole 10 (325ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 10 (325ft) Statistics by Player Rating
6
Value (Y) axis
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

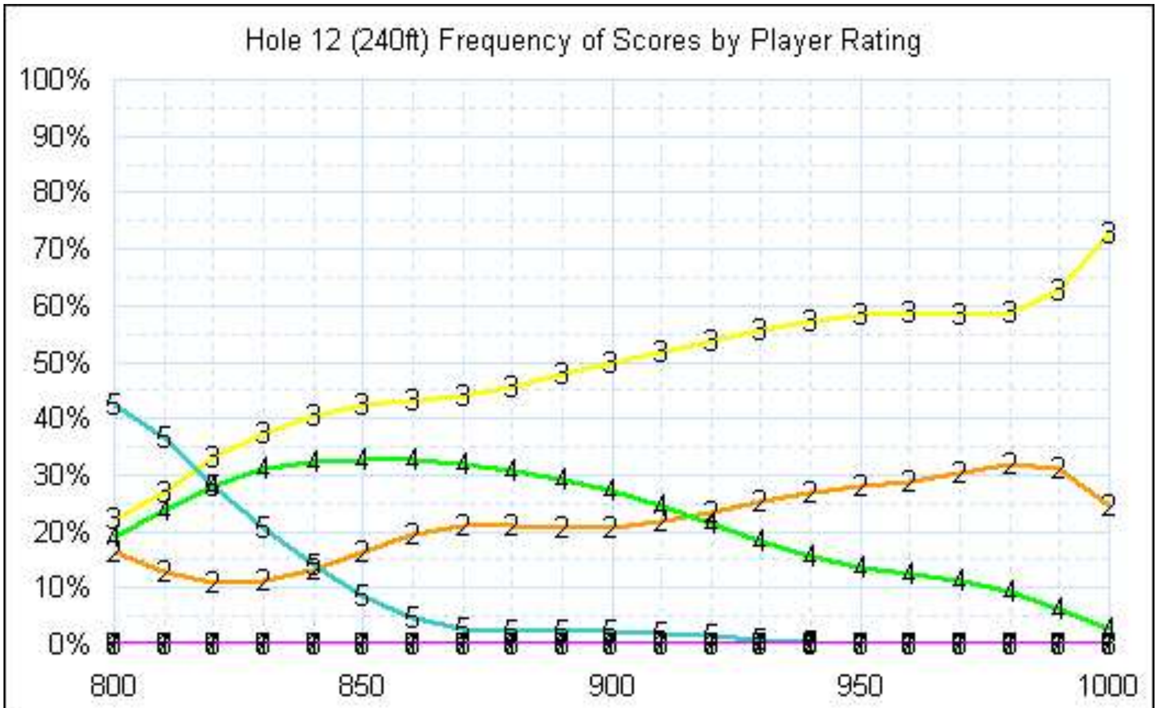
Hole 10

-

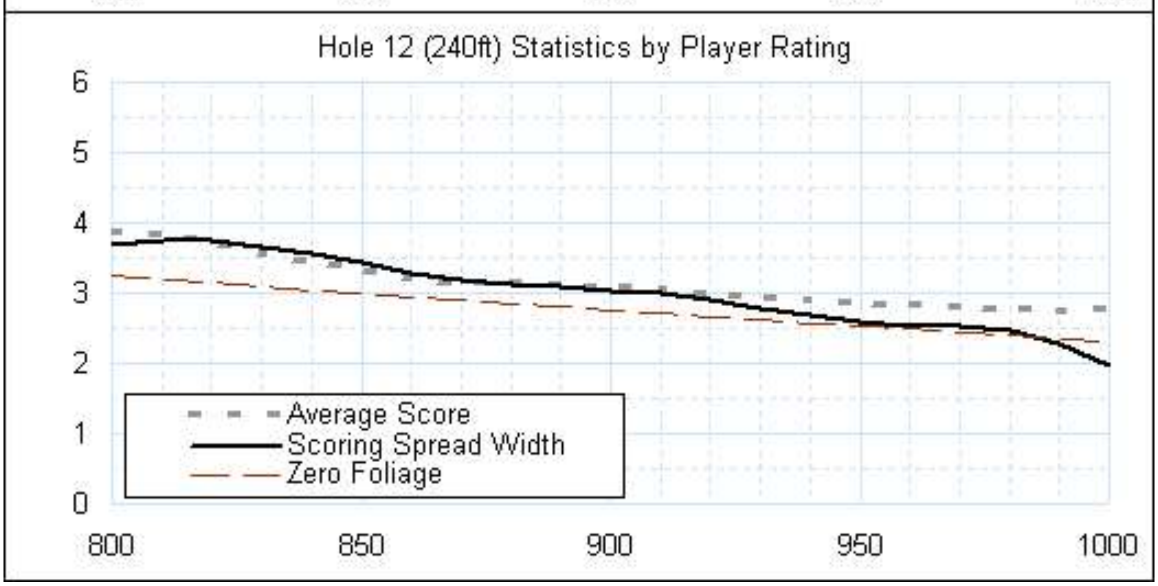

Hole 11 (625ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 11 (625ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

Hole 11

-



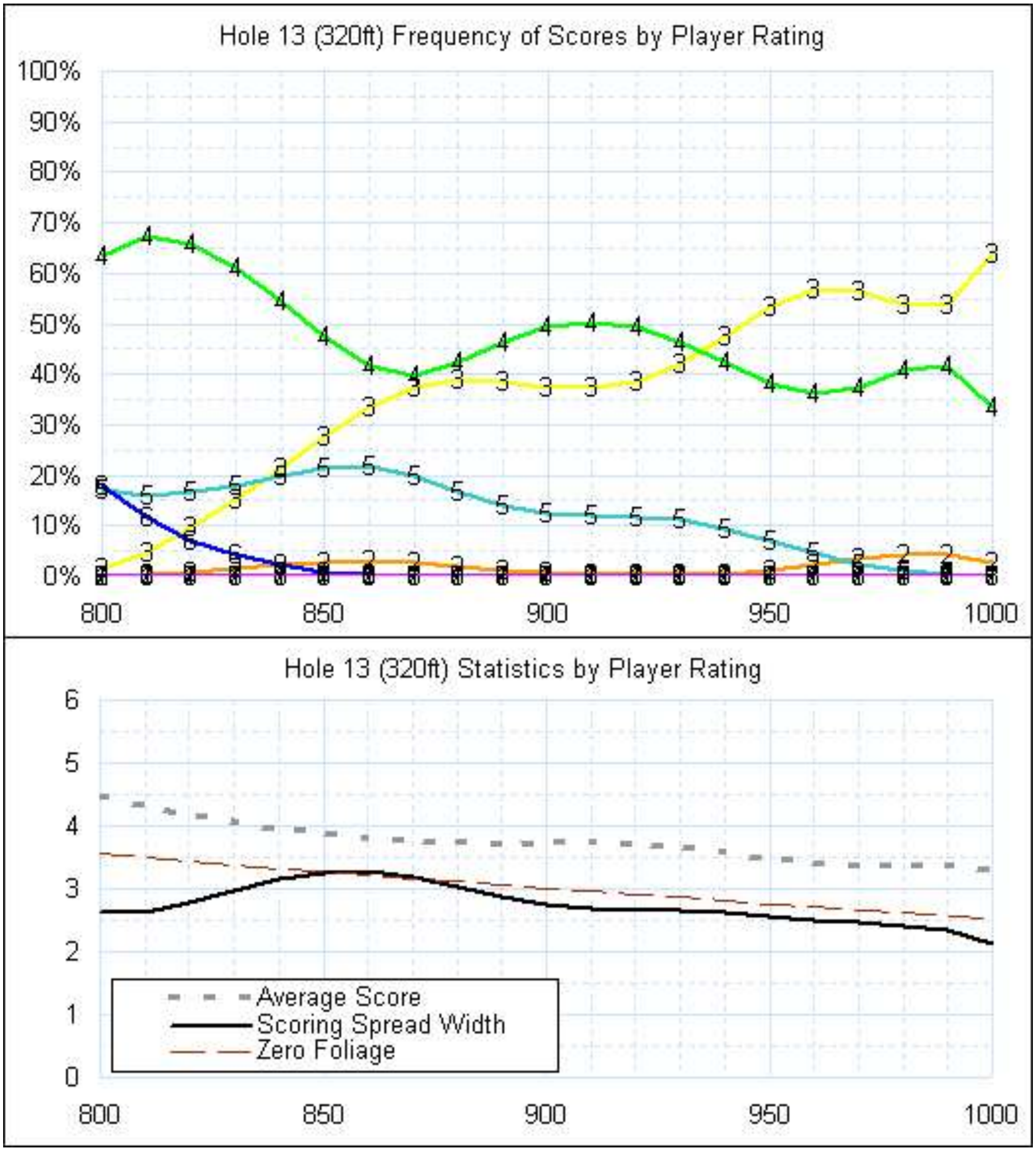
Hole 12 (240ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000



Hole 12 (240ft) Statistics by Player Rating
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 12

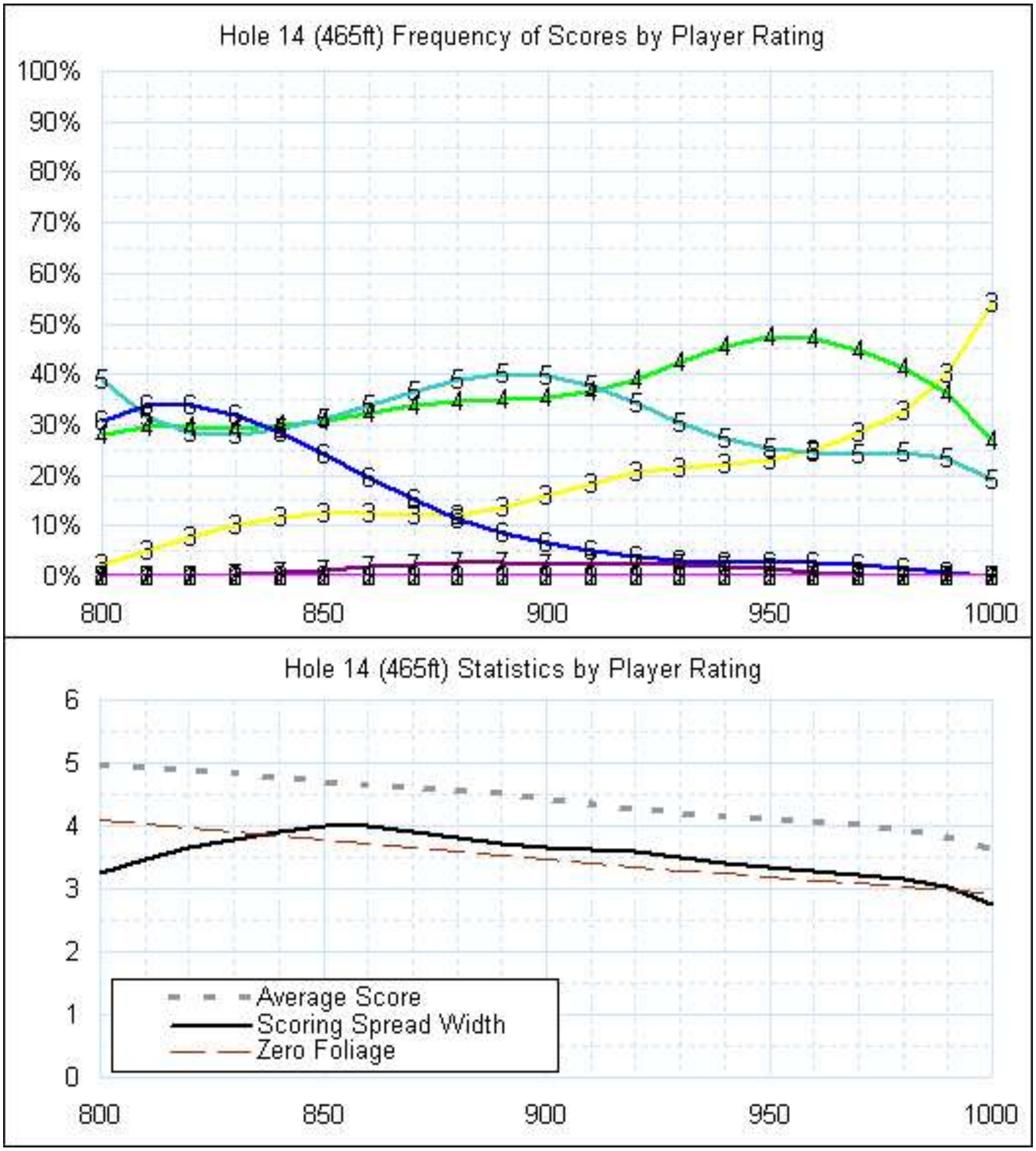
-



Hole 13 (320ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 13 (320ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

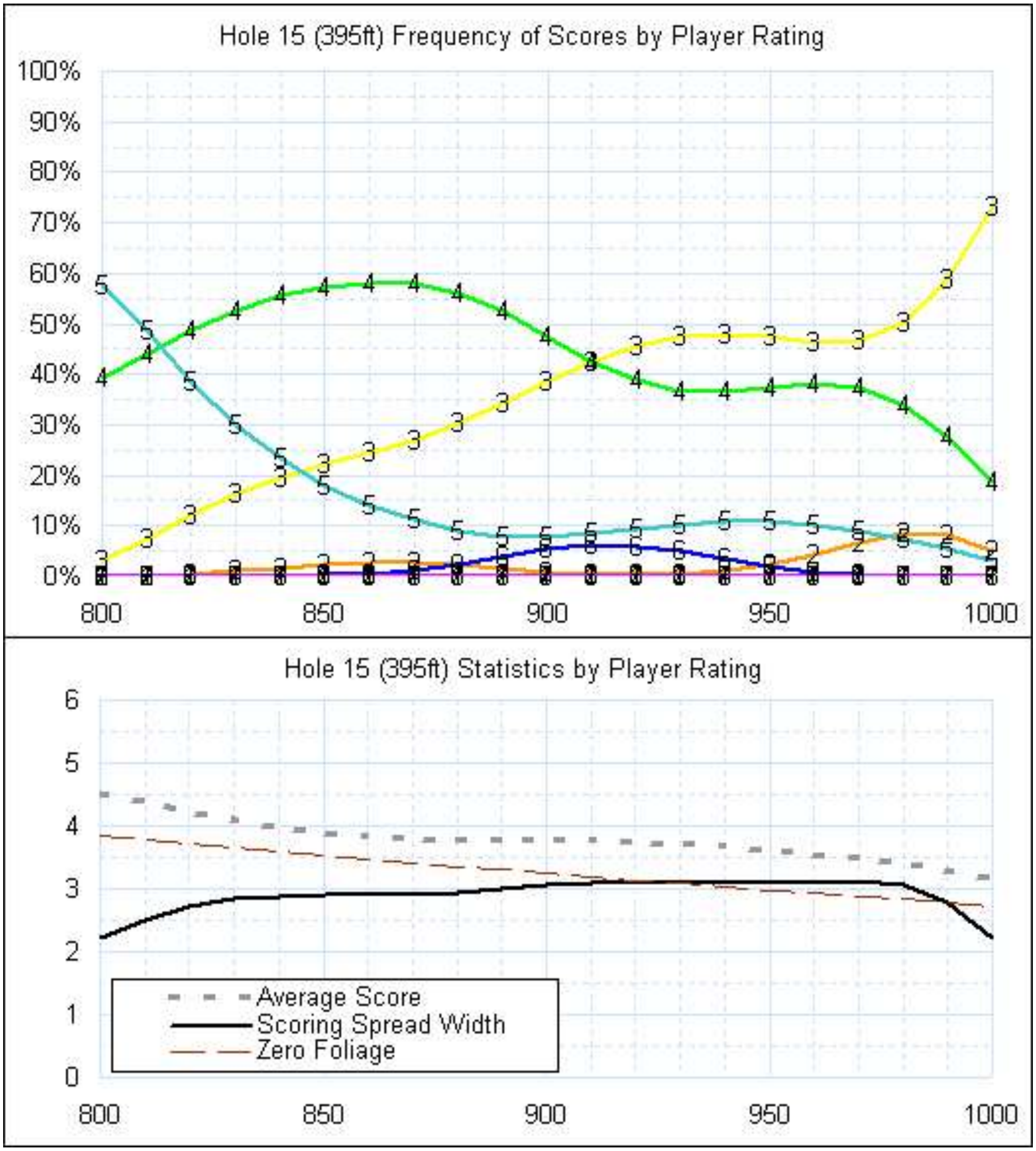
Hole 13

-


Hole 14 (465ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 14 (465ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

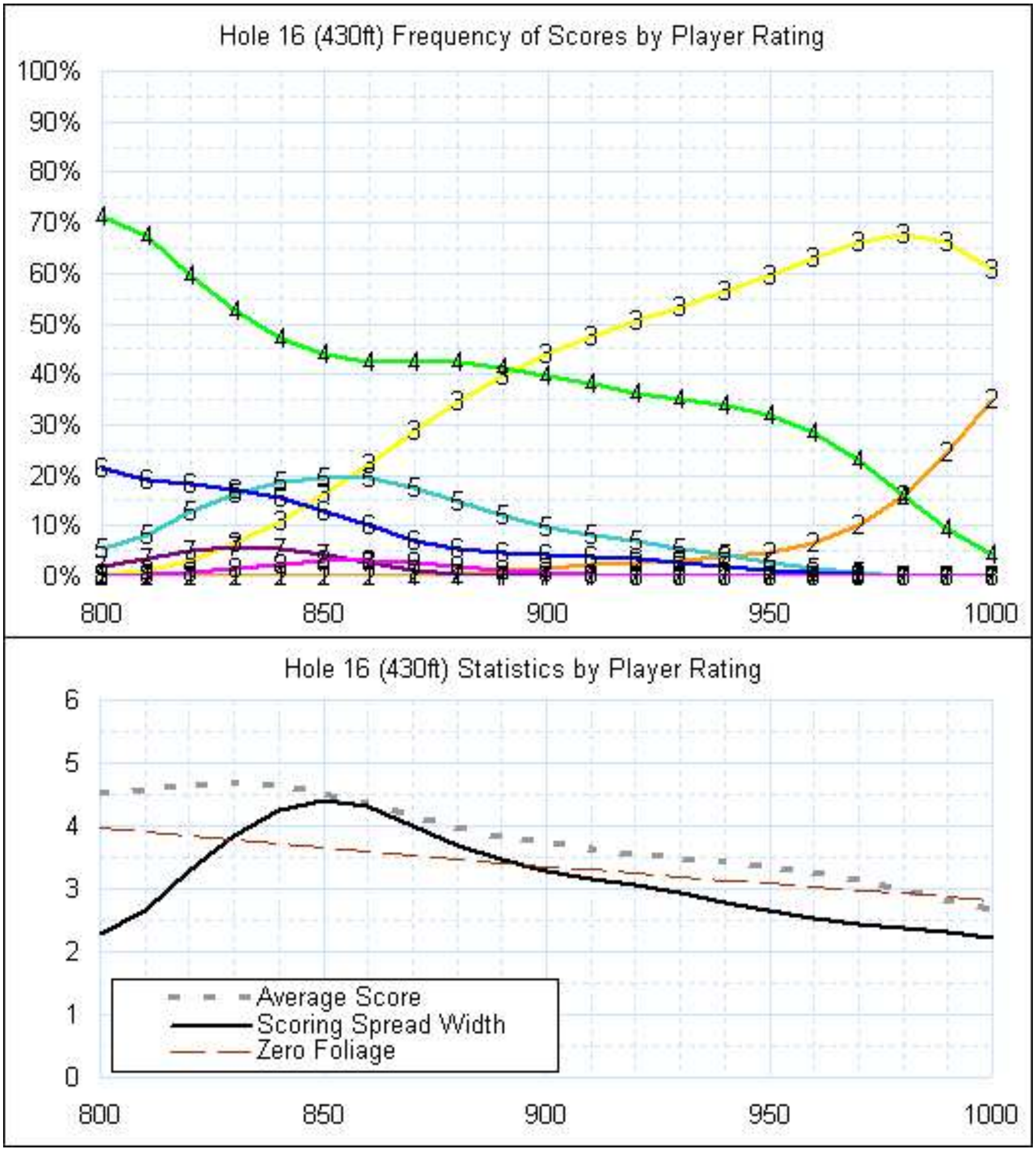
Hole 14

-


Hole 15 (395ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 15 (395ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 15

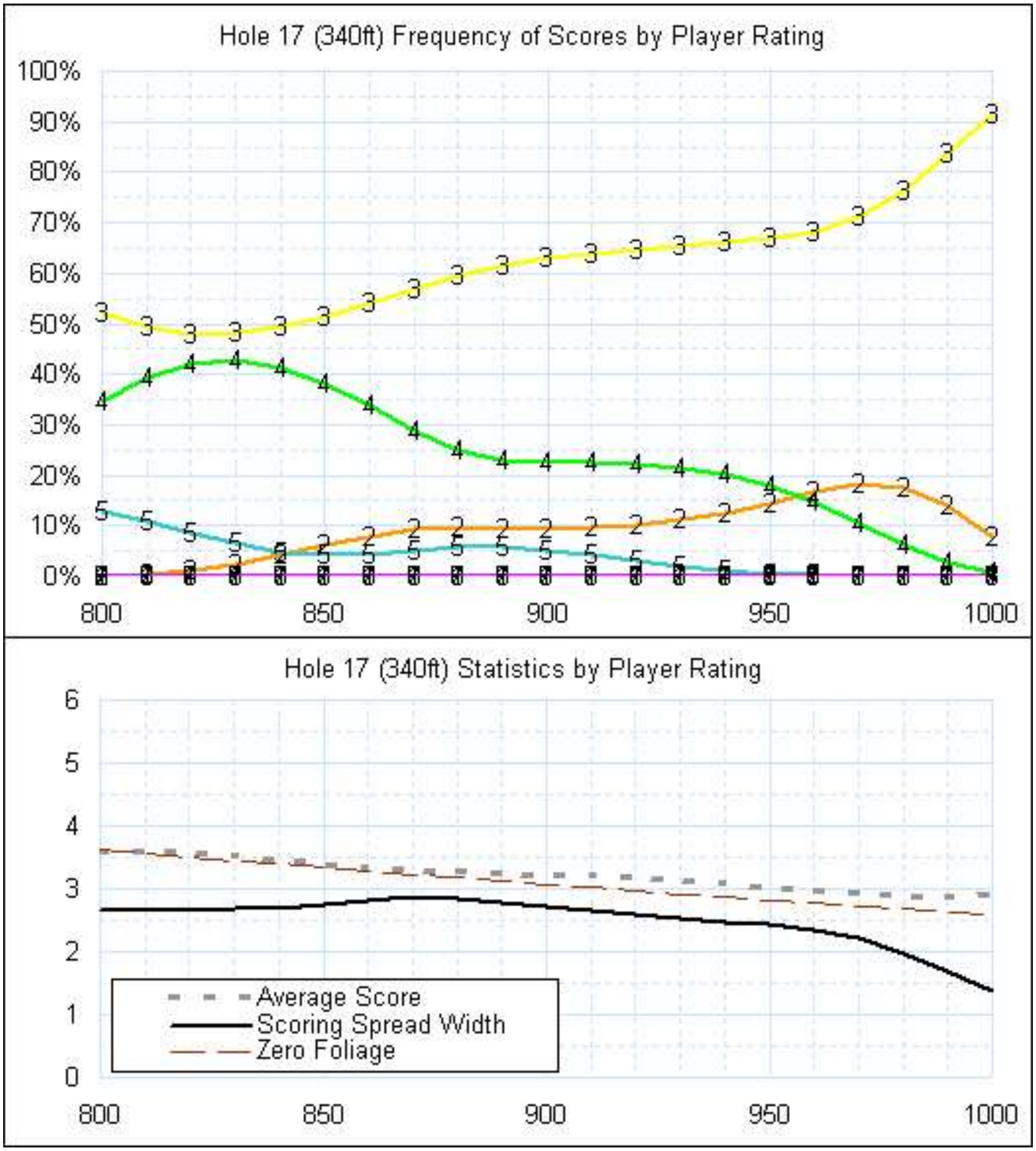
-



Hole 16 (430ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 16 (430ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 16

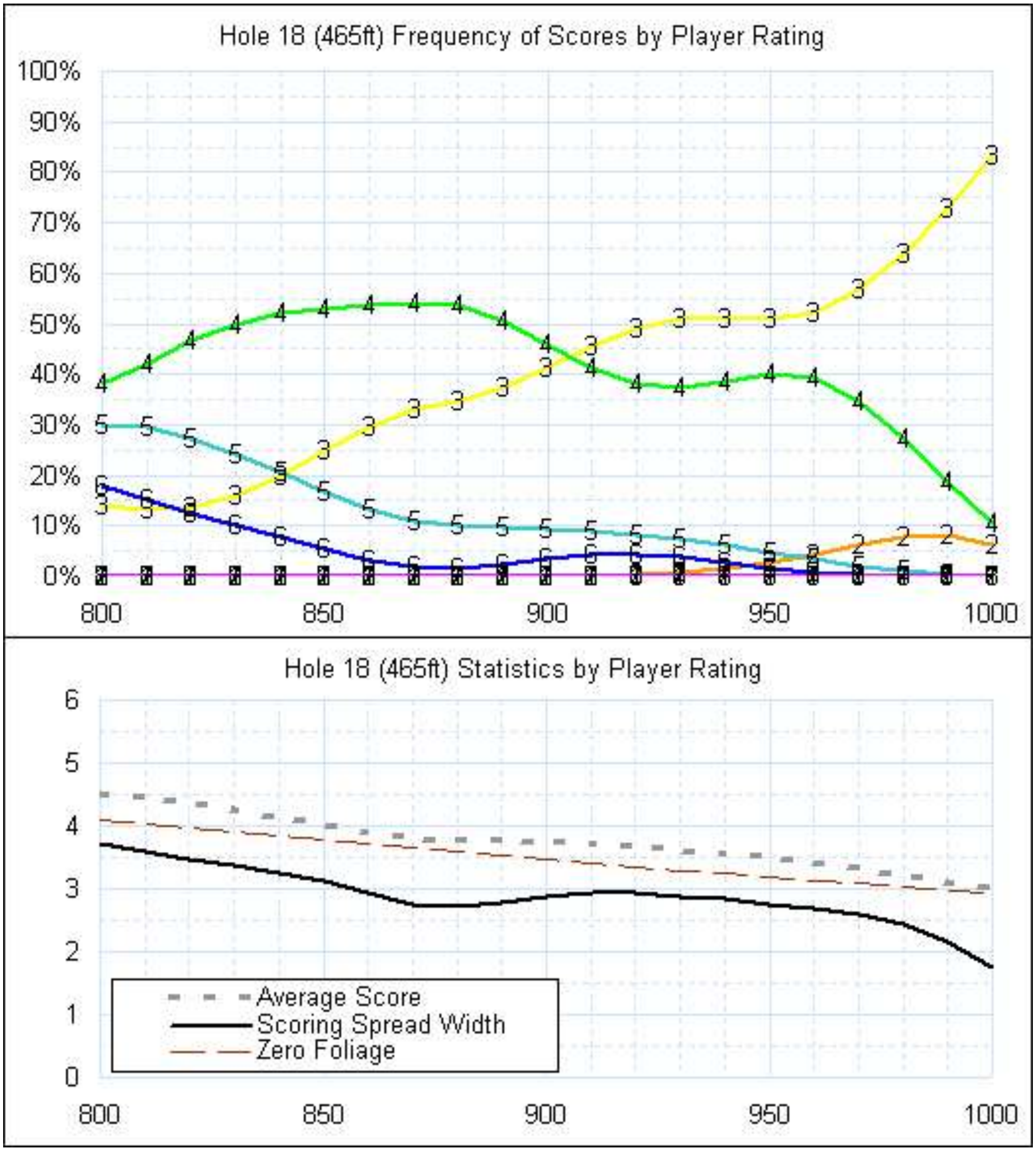
-



Hole 17 (340ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 17 (340ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage
800
850
900
950
1000

Hole 17

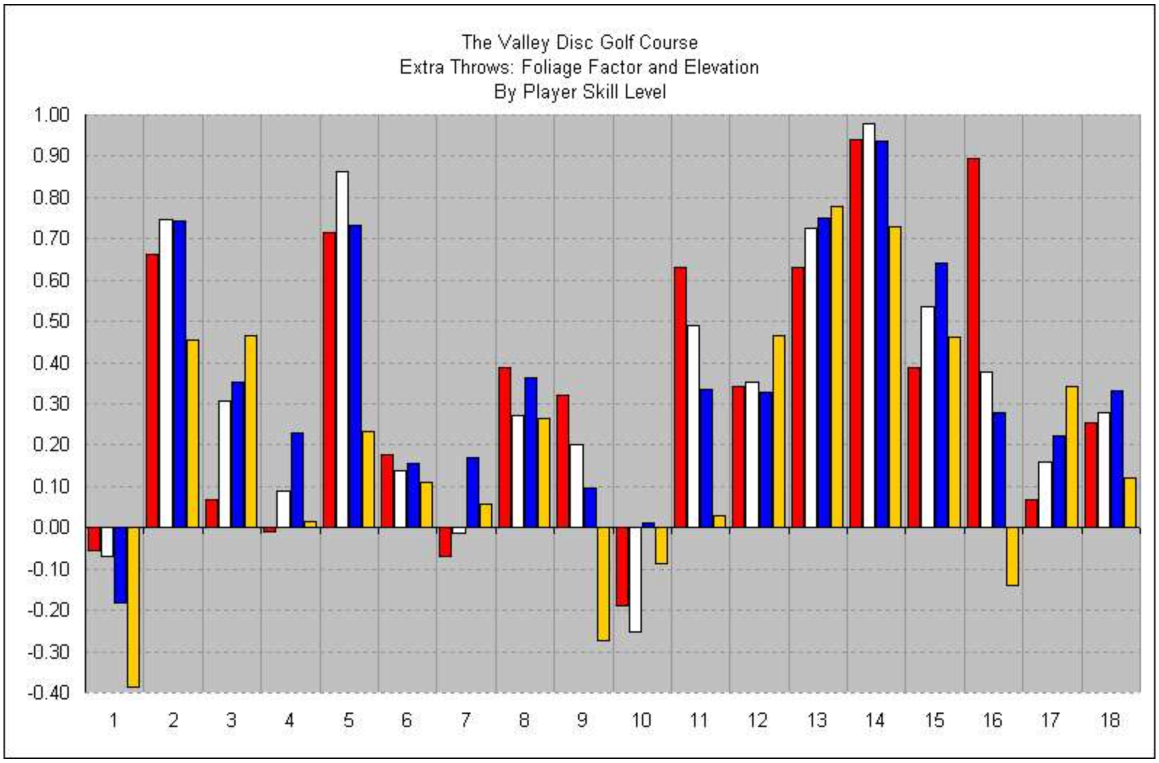
-


Hole 18 (465ft) Frequency of Scores by Player Rating
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
800
850
900
950
1000
Hole 18 (465ft) Statistics by Player Rating
6
5
4
3
2
1
0
Average Score
Scoring Spread Width
Zero Foliage

Hole 18

Extra Throws

As a reward for reading all the way through, here is a graph comparing the average scores to the theoretical "zero foliage" scores from the Hole Forecaster. The bars show how much harder (or, if negative, easier) each hole is compared to a flat, wide-open hole. Bars are color-coded by skill level. Red = 850 rating, White = 900, Blue = 950, Gold – 1000.



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Note that the combination of effective length and foliage factor does not have the same impact for different skill levels. For example, Hole 16, being downhill, plays easy for 1000-rated players. However, lower-skilled players find a lot of trouble to deal with.

Conversely, Hole 3, a short hole with a screen of trees to get through, plays "normal" for players who would usually score a 3 on a hole of this length anyway. But, those trees offer small enough gaps to mess with the 1000-rated players expectations of getting mostly 2's.