**Fun with Numbers from**
**the MA1**
**2012 PDGA Amateur World Championships**
**at the**
**Plantation Ruins Course**

November 20, 2012
Steve West Disc Golf, LLC

Tom Usselman provided the hole-by-hole-by-player scores from the 2012 PDGA Amateur World Championships for Advanced Players at the Plantation Ruins Disc Golf Course. Following are some statistics that can be generated.

Players to focus on

I chose to use figures for the entire field. This is appropriate if the job of the course is to inform all the players about how they stack up against all other players.

For other purposes, it may be appropriate to look at subsets. For example, if the purpose of a course is to allocate prizes, then the top third of the players should be the focus. If the course is being fined- tuned for a specific skill level, then a set of players that average that rating should be looked at.

Scoring Spread

The main purpose of this paper is to show how the statistic "Scoring Spread" can be used to analyze how well a disc golf course did its job. So, some illumination of Scoring Spread is appropriate.

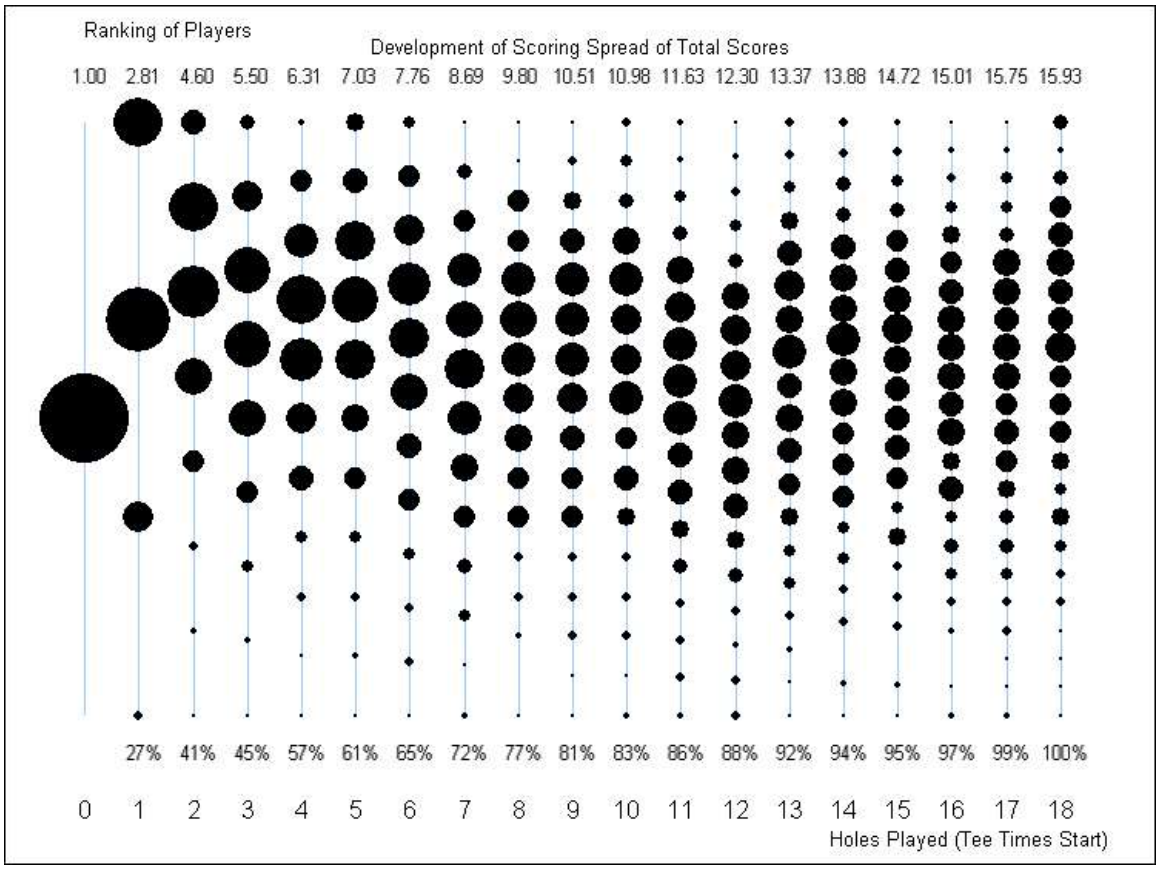
Scoring Spread is derived from Information Theory, and is related to the Entropy of a distribution. A bigger (wider) Scoring Spread gives more players more information about how they stack up against other players.

The more different scores, and the more evenly they are distributed, the higher the Scoring Spread.

At one extreme, if all players are tied at the same score, the Scoring Spread is 1.00. At the other extreme, if every player has a unique score, then the Scoring Spread is equal to the number of players.

Graphic Example of Scoring Spread

As more information goes into a system, the Scoring Spread widens. Here is a chart showing how the ranking of the players changes as they play more holes.




Before any holes area played, everyone is tied, so the Scoring Spread is just 1.00 (See the row of numbers across the top.)

After everyone plays Hole 1, all players have one of four different scores: the players at the top have scored a 2, more players got a 3, a few 4, and a handful 5. If these scores were evenly distributed among the players, the Scoring Spread would be 4.00. However, because the scores are concentrated (mostly at 2 and 3), the Scoring Spread is narrower at 2.81.

After everyone plays Hole 2, the total scores range from 4 to 11. However, most of the scores are really between 4 and 8. The Scoring Spread is 4.60.

Note the progression after Holes 8, 9, and 10 are played. The range of total scores remains the same (the best score is 15 throws lower than the worst). The Scoring Spread increases because the total scores become more evenly distributed among the players.

For example, the gap in scores after Hole 8 is filled in after Hole 9. Also, there is less concentration at the 6th, 7th, and 8th best scores.

Similarly, for the last 3 holes, there is a range of 21 throws from the best score to the worst. The widening of the Scoring Spread comes from the evening-out of the distribution of scores.

The widest possible Scoring Spread for 281 players would be 281; every player would have their own unique score. However, that is not possible with a mere 18 holes that give out just a few little scores each. If every hole gave out the best score to the best player, etc. and the worst score on each hole went to the worst player, scores would range from 36 to 100, and the Scoring Spread would be 25.16. The Scoring Spread would not be 65.00 because not every total score within the range of 36 to 100 would be generated. Also, some of the scores (58, for example) would go to many players.

Also shown (across the bottom) is the correlation of the running total score to the final total score. Correlation is sometimes used as a measure of how well a course is doing its job, so I've included a few comparisons.

Hole by Hole Scoring Distributions

One measure of whether a disc golf hole is doing its job is whether is gives out a range of scores. A hole where everyone scores a 3 does nothing to help find who the better players are.

One rule of thumb is that no more than 2/3 of the field should get the same score. Scoring Spread is a more refined version of this rule of thumb. For example, a hole that gives out 1/3 2's and 2/3 3's would be "OK" according to the rule of thumb. Also "OK" would be a hole that gives out 1/6 2's, 2/3 3's and 1/6 4's. However, we know that the second hole is better at sorting out players. Scoring Spread quantifies how much better. The Scoring Spread of 1/3, 2/3 is 1.89, and the Scoring Spread of 1/6, 2/3, 1/6 is 2.38.

Here are the single-hole Scoring Spreads, along with the scoring distribution[1] (which is the % of players that received each score):

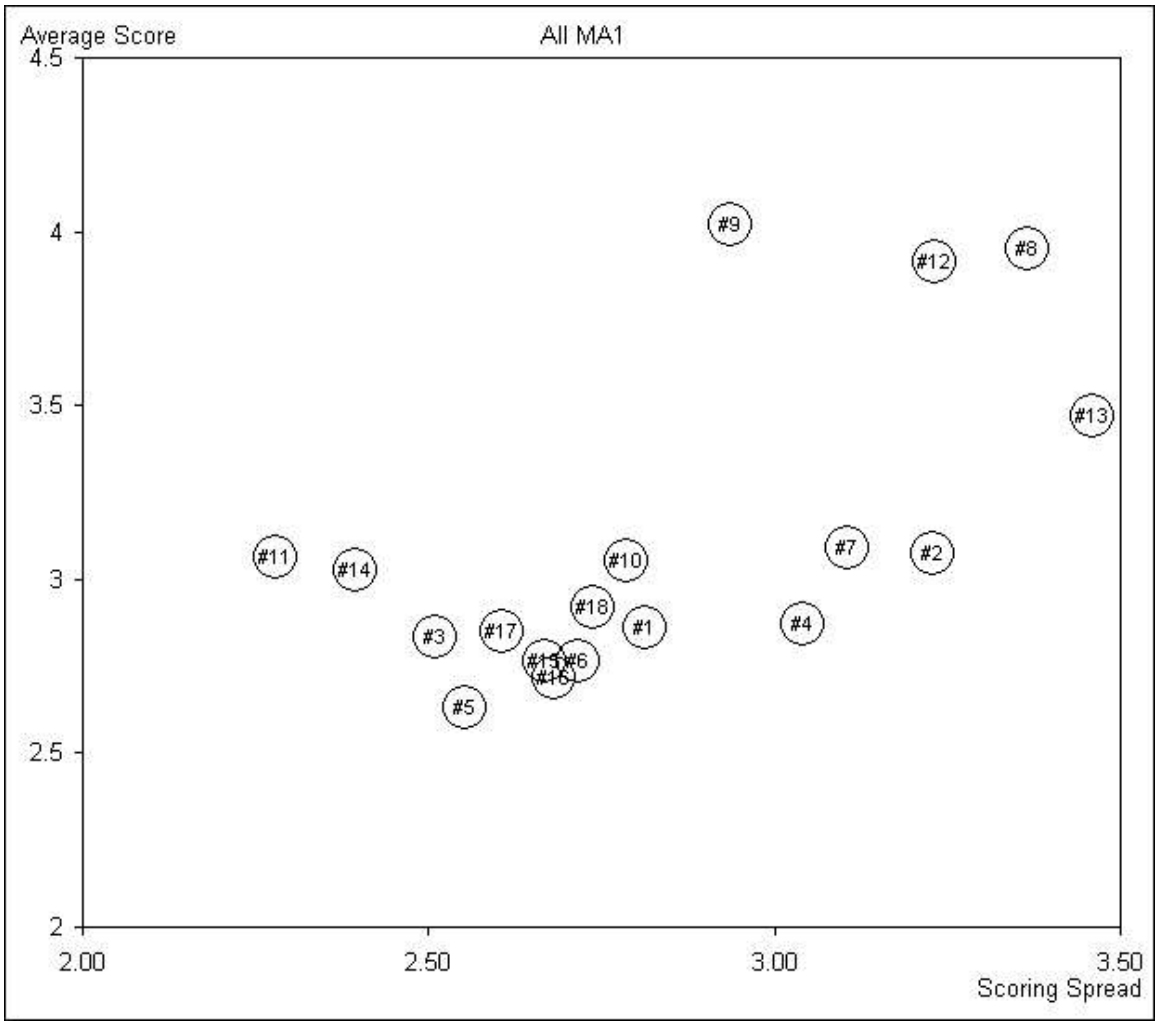
| Hole | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Scoring Spread: |
|---|---|---|---|---|---|---|---|---|---|
| #1 | | 31% | 54% | 13% | 2% | | | | 2.81 |
| #2 | | 23% | 54% | 18% | 3% | 2% | | | 3.23 |
| #3 | 0.4% | 27% | 63% | 9% | 1% | | | | 2.51 |
| #4 | | 34% | 47% | 16% | 3% | | | | 3.04 |
| #5 | | 46% | 47% | 7% | 1% | | | | 2.55 |
| #6 | | 36% | 53% | 10% | 1% | 0.4% | | | 2.72 |
| #7 | | 21% | 53% | 22% | 4% | | | | 3.11 |
| #8 | | | 33% | 47% | 13% | 5% | 1% | 0.4% | 3.37 |
| #9 | | 0.4% | 21% | 59% | 17% | 3% | 0.4% | | 2.94 |
| #10 | | 19% | 60% | 19% | 2% | | | | 2.79 |
| #11 | | 11% | 73% | 14% | 2% | | | | 2.28 |
| #12 | | 0.4% | 32% | 47% | 17% | 2% | 1% | | 3.23 |
| #13 | | 9% | 49% | 31% | 7% | 4% | | | 3.46 |
| #14 | | 15% | 70% | 13% | 2% | | | | 2.39 |
| #15 | | 36% | 53% | 10% | 1% | | | | 2.67 |
| #16 | | 40% | 49% | 10% | 1% | | | | 2.68 |
| #17 | | 28% | 60% | 10% | 2% | | | | 2.61 |
| #18 | | 26% | 59% | 14% | 2% | | | | 2.74 |

All of these Scoring Spreads look acceptable on their face, and some are notably wide. See Hole #13, for example.

[1] In a totally unrelated note, my preferred definition of Par would set par to 2 for holes 5 and 16. If 4 out of ten players can get a 2, then that indicates the "errorless" way to play the hole is to get a 2.

Scoring Spread vs. Average Score

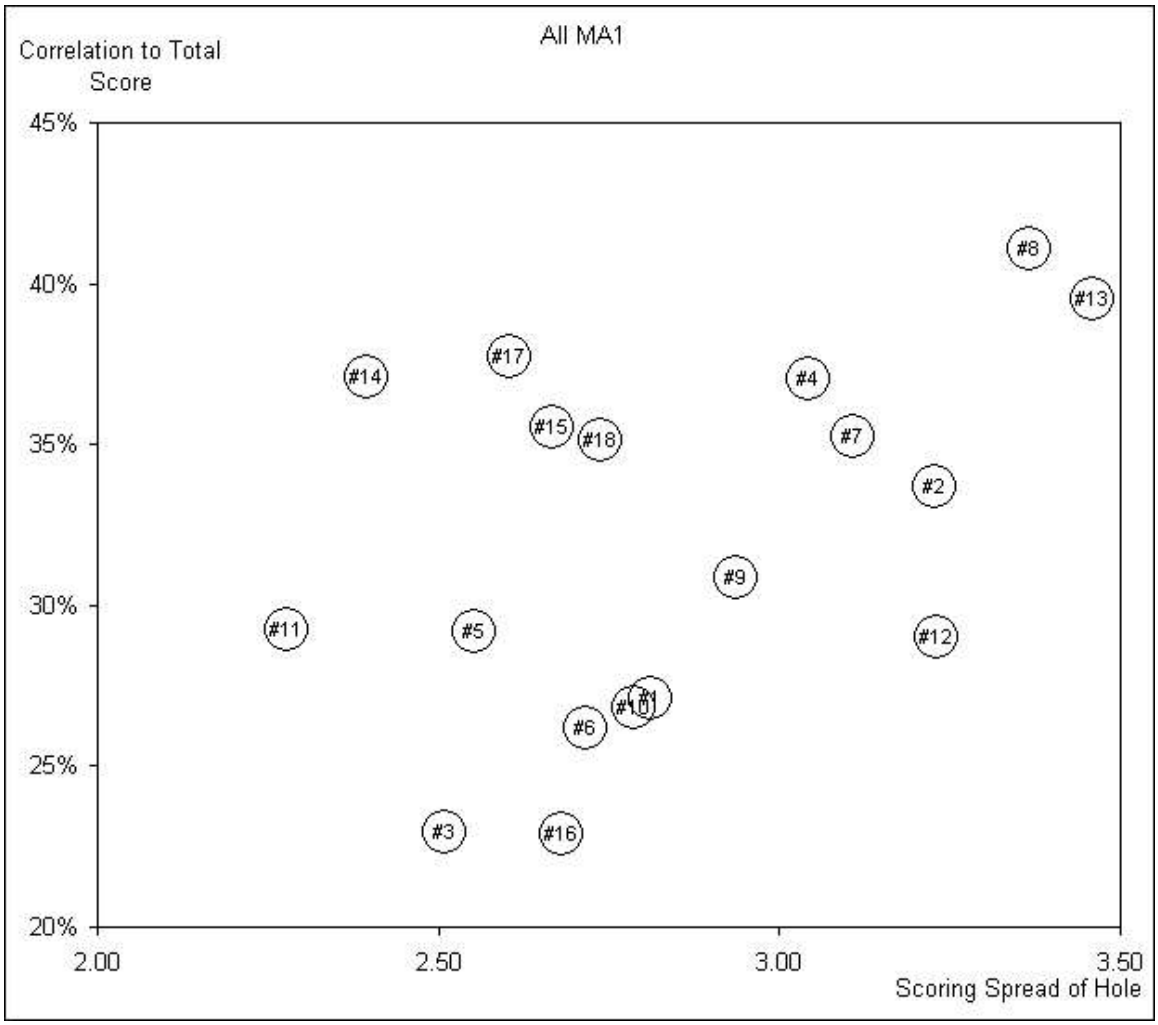
One interesting comparison is Scoring Spread vs. Average Score. Higher-scoring holes have an easier time generating wider Scoring Spreads, so holes that generate a wider Scoring Spread without relying on a higher average score deserve special notice.




Note Hole #13 achieved its wide Scoring Spread in spite of not having the highest average score.

Correlation vs. Scoring Spread

Here is a chart showing the correlation of single-hole scores to the total score, plotted against Scoring Spread. The point is that Correlation and Scoring Spread are pointing in the same general direction as to where "good" holes should be.



Contribution to the Scoring Spread of Total Scores

While single-holes should have wide Scoring Spreads, something even more important is how much those holes contribute to the widening of the Scoring Spread of Total Scores.

For example, if there were an evil hole that decided to give out bad scores to good players, that hole would cancel out some of the effects of the other holes. The Scoring Spread for Total Scores would get narrower. The wider the Scoring Spread on the evil hole, the more damage it could do.

To measure the contribution of single-holes, I calculated the Scoring Spread of Total Scores with and without the score from each hole. The Contribution is the difference.

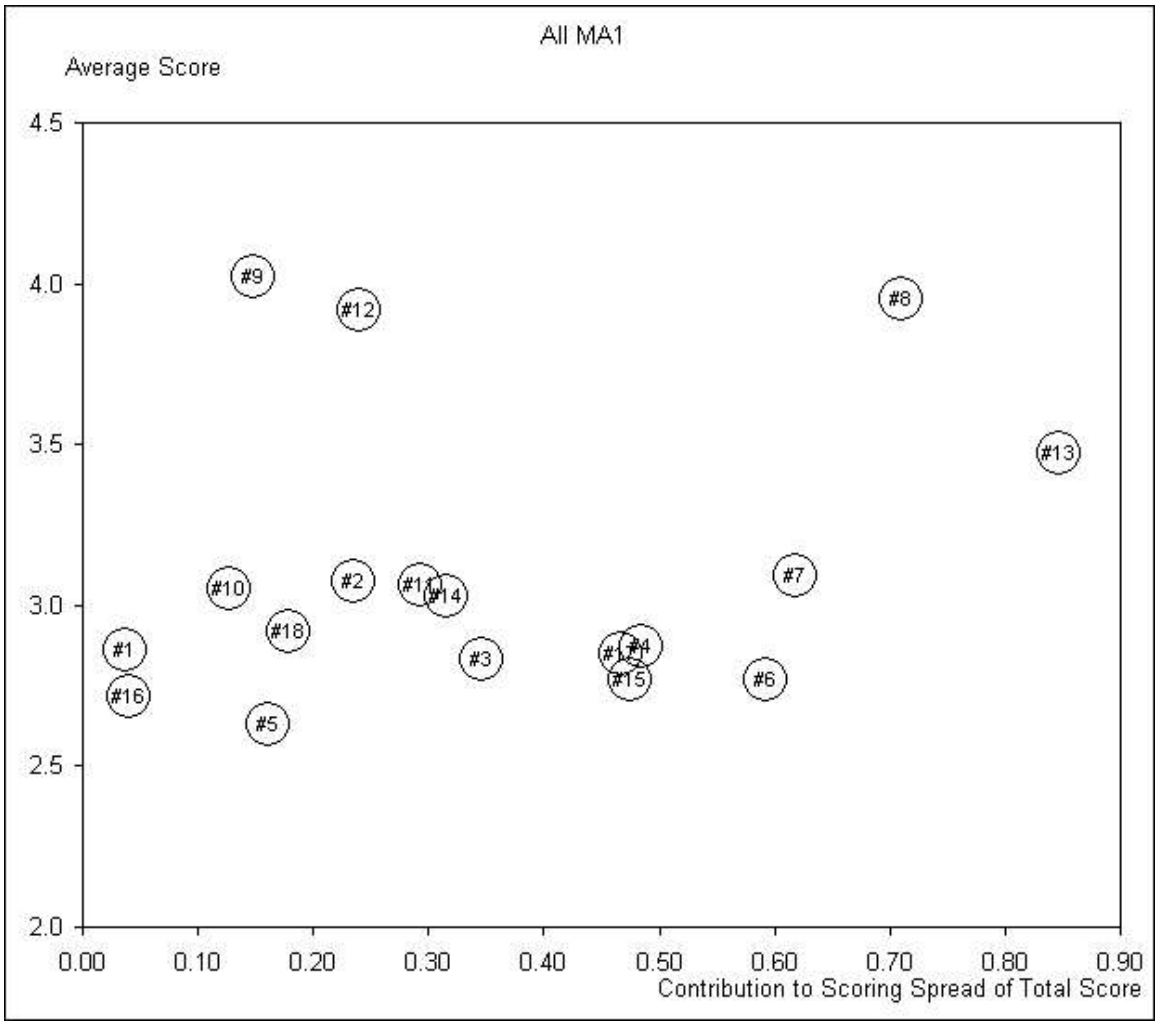
| Hole | Contribution |
|---|---|
| #1 | 0.04 |
| #2 | 0.23 |
| #3 | 0.35 |
| #4 | 0.48 |
| #5 | 0.16 |
| #6 | 0.59 |
| #7 | 0.62 |
| #8 | 0.71 |
| #9 | 0.15 |
| #10 | 0.13 |
| #11 | 0.29 |
| #12 | 0.24 |
| #13 | 0.85 |
| #14 | 0.32 |
| #15 | 0.48 |
| #16 | 0.04 |
| #17 | 0.47 |
| #18 | 0.18 |

Again, Hole #13 comes out ahead. At the other extreme, the Scoring Spread of Total Scores would have been almost the same had Hole #1 or Hole #16 not been played.

It is notable that all the Contributions are positive for this course. It is possible - not even that unlikely - that a hole or two would make a small negative contribution.

Even thought Holes #11 and #14 have the narrowest Scoring Spreads, their Contribution is in the middle.

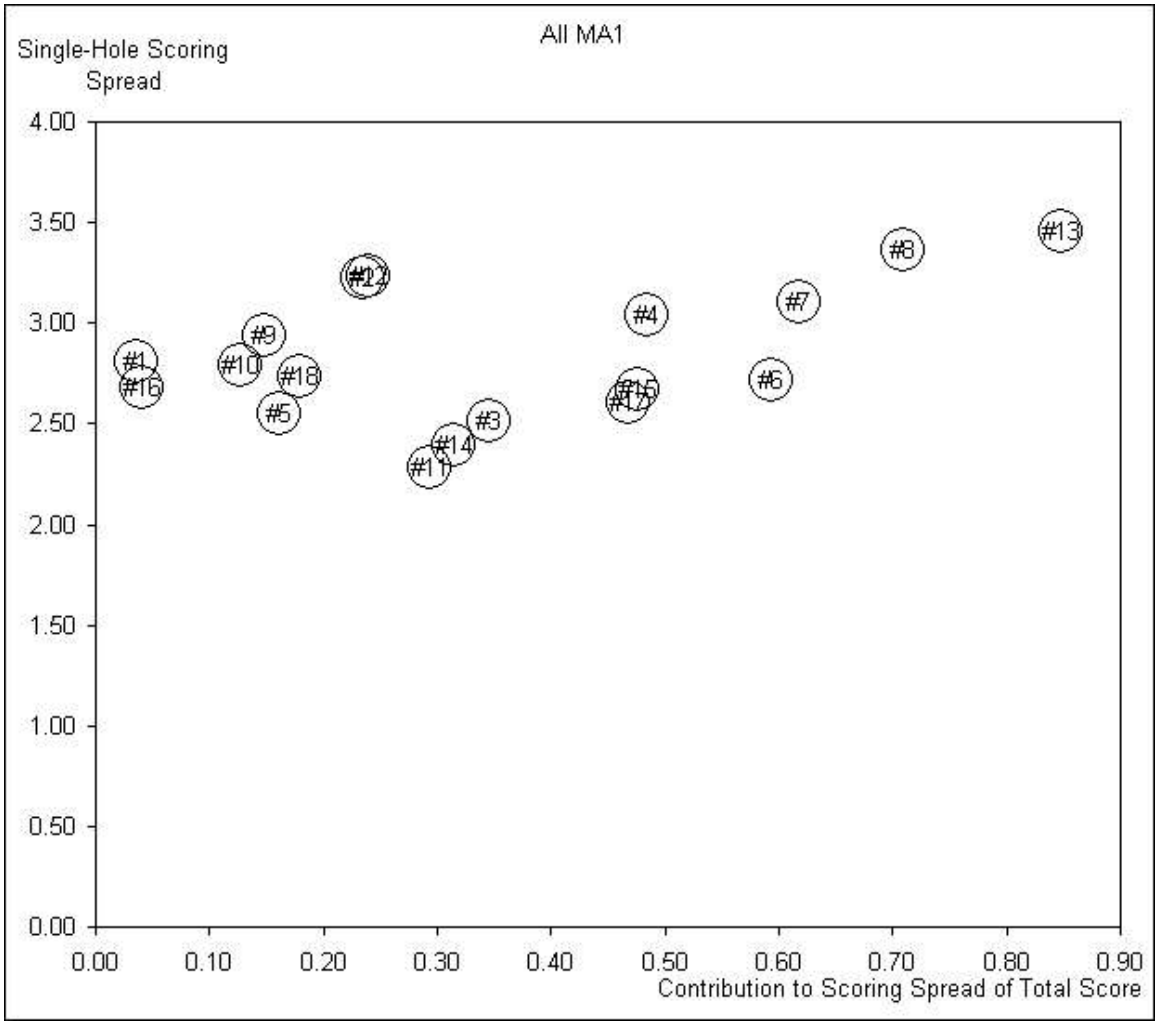
Contribution vs. Average Score




Hole #13 still looks good, but Hole #9 is not meeting the expectations one might have for a hole with such a high average score and single-hole Scoring Spread.

Contribution to Scoring Spread of Total Scores vs. Single-Hole Scoring Spreads

To take a closer look at the interaction between single-hole Scoring Spreads and the Contribution to the Scoring Spread of Total Scores.




Hole #13 is using its wide single-hole Scoring Spread to good advantage, but Hole #1 and #16 are contributing almost nothing, even though they don't have the narrowest single-hole Scoring Spreads.

Contribution Compared to Randomly Assigning Scores

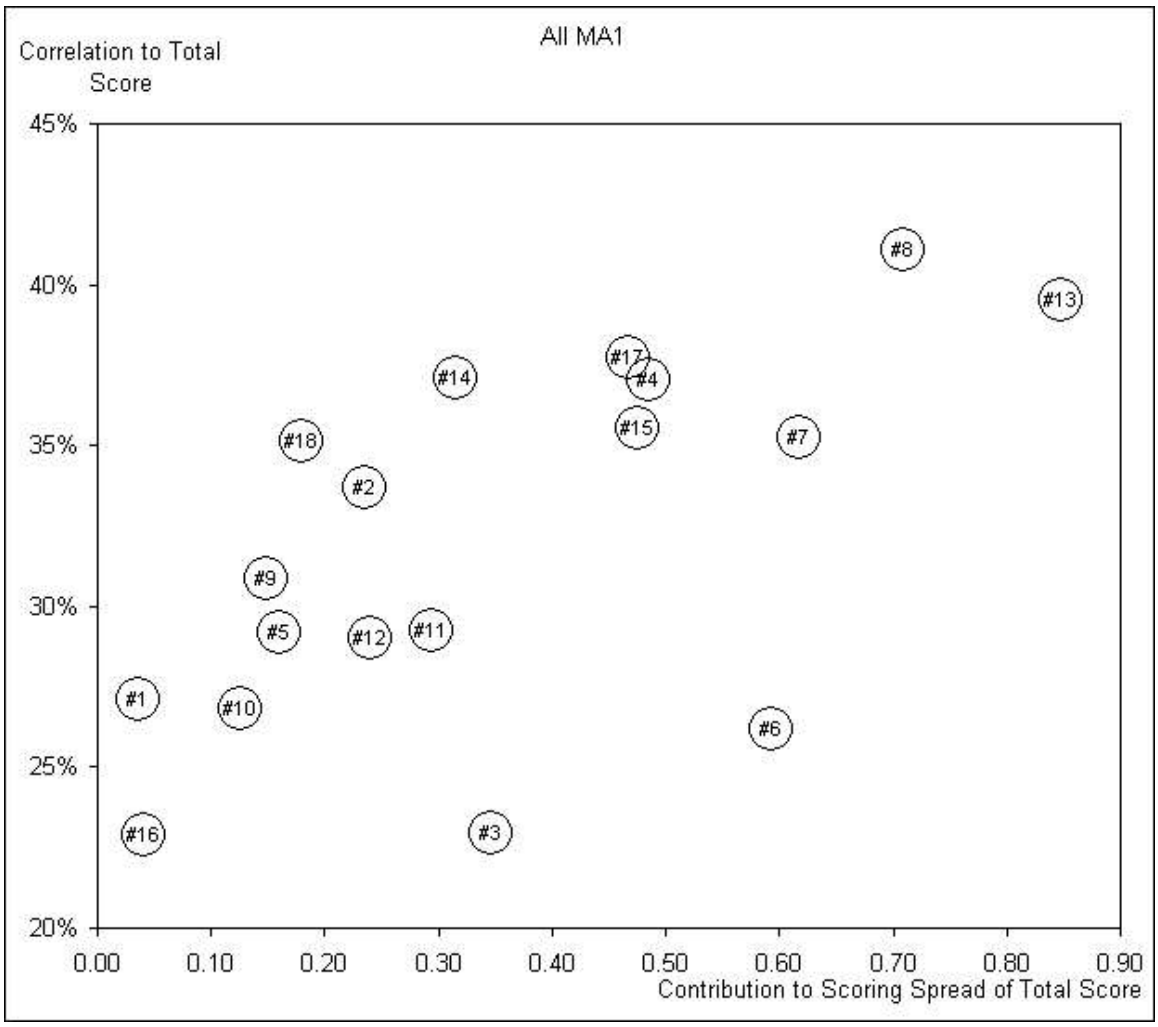
The contribution of a hole depends on two things: the scores it has available to hand out, and the way it hands them out. To see how well each hole is assigning the scores to players, I computed the Contribution that each hole would make if that hole's set of scores were randomly distributed to players. Then, I compared the actual contribution of each hole to the range of possible contributions that would result from randomly rearranging the single-hole scoring distributions.

| Hole | Low 2.5% | High 2.5% | Actual Contribution | Better than |
|---|---|---|---|---|
| #1 | -0.62 | 0.38 | 0.04 | 73% |
| #2 | -0.66 | 0.47 | 0.23 | 87% |
| #3 | -0.68 | 0.30 | 0.35 | 98% |
| #4 | -0.39 | 0.60 | 0.48 | 93% |
| #5 | -0.46 | 0.52 | 0.16 | 69% |
| #6 | -0.56 | 0.50 | 0.59 | 98.9% |
| #7 | -0.68 | 0.35 | 0.62 | 99.76% |
| #8 | -0.34 | 0.90 | 0.71 | 91% |
| #9 | -0.66 | 0.34 | 0.15 | 88% |
| #10 | -0.74 | 0.24 | 0.13 | 93% |
| #11 | -0.49 | 0.44 | 0.29 | 91% |
| #12 | -0.62 | 0.41 | 0.24 | 90% |
| #13 | -0.67 | 0.46 | 0.85 | 99.99% |
| #14 | -0.23 | 0.73 | 0.32 | 60% |
| #15 | -0.58 | 0.30 | 0.48 | 99.5% |
| #16 | -0.64 | 0.28 | 0.04 | 82% |
| #17 | -0.32 | 0.70 | 0.47 | 85% |
| #18 | -0.51 | 0.43 | 0.18 | 81% |

Again, Hole #13 shines, doling out its scoring distribution in a way that is better than almost any other possible way. All the holes were better than the average result of randomly assigned scores.

Contribution to Scoring Spread of Total Scores vs. Correlation

Again, to "ground" the new statistic, here is a comparison to Correlation.




Again, this shows they are trying to measure something similar and Contribution actually agrees more with Correlation than does single-hole Scoring Spread.

Further Uses for Scoring Spread

Just as we can compute the Contribution to Scoring Spread of Total Scores for single-holes, we can also compute the contributions of pairs of hole (or any combinations, for that matter.)

The top ten pairs of holes are:

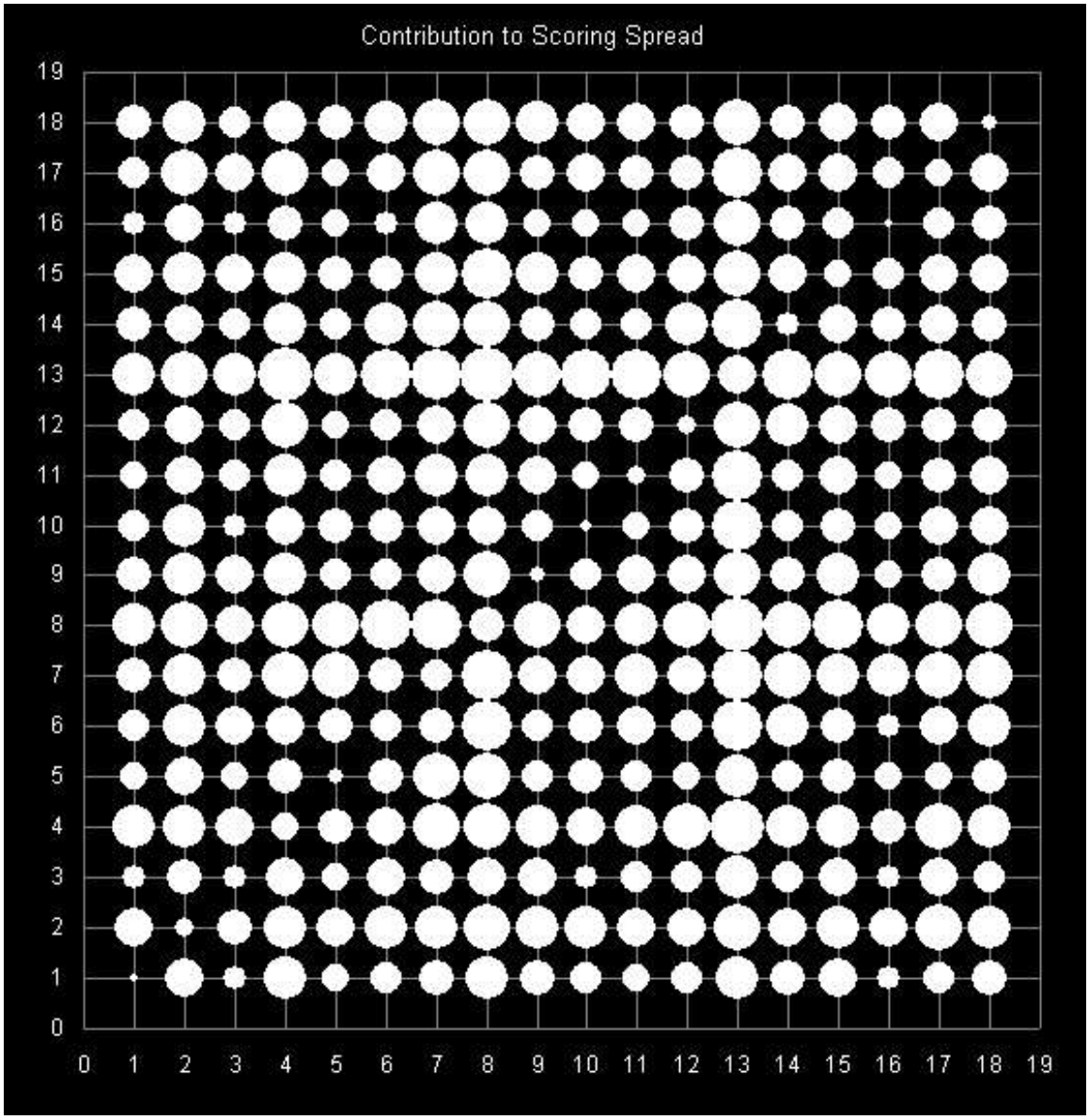
| Holes | Contribution |
|---|---|
| #8 & #13 | 1.72 |
| #4 & #13 | 1.70 |
| #11 & #13 | 1.60 |
| #7 & #13 | 1.58 |
| #8 & #15 | 1.56 |
| #7 & #8 | 1.55 |
| #13 & #17 | 1.53 |
| #6 & #13 | 1.52 |
| #6 & #8 | 1.51 |
| #13 & #14 | 1.49 |

Note that all of these are a result of one hole being combined with one of the strongest holes (#8 or #13).

And the bottom ten are:

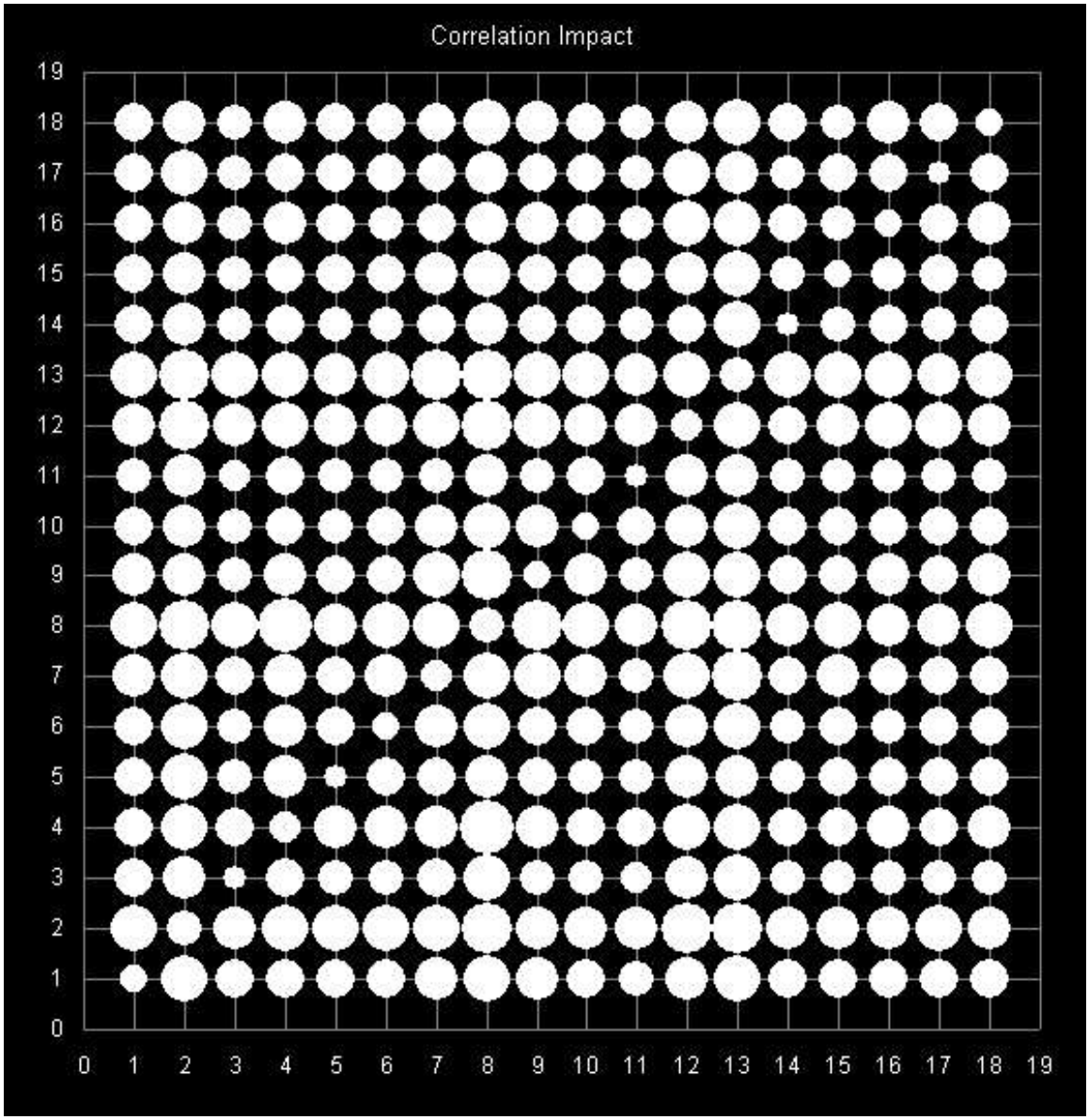
| Holes | Contribution |
|---|---|
| #10 & #16 | 0.50 |
| #1 & #5 | 0.47 |
| #5 & #12 | 0.44 |
| #11 & #16 | 0.44 |
| #1 & #11 | 0.42 |
| #6 & #16 | 0.39 |
| #3 & #16 | 0.39 |
| #1 & #16 | 0.39 |
| #3 & #10 | 0.38 |
| #1 & #3 | 0.38 |

Here is a chart showing the relative contributions of pairs of holes (the diagonal is the contribution of single-holes).



Correlations of Pairs of Holes

For comparison, here is a similar chart based on the correlation of scores from pairs of holes to total scores.



Synergy Between Holes

Some combinations of holes generate more of a Contribution to the Scoring Spread of Total Scores than would be expected based on their single-hole contributions. On the other hand, some combinations produce less than expected. The exceptional cases at both ends might be interesting to look at, to see if there are some characteristics of the holes that may explain the synergy (or lack thereof).

The top ten are:

| Holes | Synergy |
|---|---|
| #2 & #17 | 0.44 |
| #2 & #9 | 0.39 |
| #9 & #18 | 0.37 |
| #1 & #4 | 0.36 |
| #8 & #9 | 0.36 |
| #4 & #12 | 0.34 |
| #2 & #10 | 0.34 |
| #11 & #13 | 0.33 |
| #10 & #18 | 0.33 |
| #1 & #2 | 0.32 |

And the bottom ten are:

| Holes | Synergy |
|---|---|
| #3 & #16 | -0.33 |
| #1 & #3 | -0.34 |
| #4 & #6 | -0.35 |
| #5 & #17 | -0.37 |
| #6 & #15 | -0.39 |
| #6 & #12 | -0.39 |
| #3 & #10 | -0.40 |
| #3 & #7 | -0.45 |
| #6 & #16 | -0.51 |
| #6 & #7 | -0.52 |

By adding up the synergy with all other holes, we can give each hole a Synergy Score.

| Hole | Synergy Score |
|---|---|
| #1 | 0.01 |
| #2 | 0.20 |
| #3 | -0.22 |
| #4 | 0.05 |
| #5 | -0.08 |
| #6 | -0.20 |
| #7 | -0.02 |
| #8 | 0.10 |
| #9 | 0.10 |
| #10 | 0.01 |
| #11 | -0.07 |
| #12 | 0.00 |
| #13 | 0.13 |
| #14 | 0.00 |
| #15 | -0.05 |
| #16 | -0.04 |
| #17 | -0.06 |
| #18 | 0.13 |

This tells us that Holes #2, #13, and #18 play well with others. A possible explanation is that they test a skill that is not well-tested by other holes.

Holes #3 and #6 do something that narrows the Scoring Spread. Perhaps they are giving low (or high) scores to players who don't deserve them.

Possible Implications

Obviously, everything points to: "Don't mess with Hole #13".

But, which holes may need attention, and why?

Hole #14 has the narrowest Scoring Spread, but it's in the middle of the pack in terms of its Contribution to the Scoring Spread of Total Scores. However, if we note that it does only slightly better than luck at handing out its scores, we might be open to ideas to make the hole less "lucky".

Hole #11 also has a narrow Scoring Spread, and like #14 assigns the same score to more than 2/3 of the players. However, the fact that the rest of the scores are both higher and lower than the mode helps it to be a useful hole. Making it (or #14) easier or more difficult might turn it into just a two-score hole, which would narrow the Scoring Spread to 2.

Hole #3 has the next-narrowest Scoring Spread, but assigns its scores much better than random luck, which results in a good contribution to the total score. If anything, Hole #3 might be made a little more difficult to give it some higher scores to dole out. Its low Synergy Score also might make it interesting to look at to see what's going on.

Hole #5 has the third-narrowest Scoring Spread, and isn't significantly better than luck at assigning those scores, so it makes a relatively small contribution. Plus, it's a par 2. It doesn't need to be changed, but it might be worth experimenting with ways to bring its average score up.

Looking at the Contributions to Scoring Spread of Total Scores, two holes jump out. Holes #1 and #16 make very little contribution. (And #16 is a par 2.) The problem seems to be a combination of unremarkable average scores and Scoring Spreads, and that these holes are only a little better than random luck at assigning scores to players. It probably wouldn't hurt to mess around with these two.

Hole #6 by itself does not look bad. However, it has negative synergy when combined with other holes. This may be something to investigate.

What about Luck?

One potential problem with using Scoring Spread is the possibility that scores are being widened (smeared) by pure luck. It is not intuitively obvious that a wider Scoring Spread is always due to properly assigning scores according to skill.

There are several pieces of evidence that point to wider Scoring Spreads being indicative of less luck, not more.

First, total scores are sums of single-hole distributions. Sums of random distributions tend to heap a lot of numbers in the middle (the classic bell curve). Wider Scoring Spreads are achieved by flattening the distribution – more scores out on the ends, and fewer in the middle. It takes a non-random force to do this.

Second, it is commonly believed among players that "lucky" courses compress scores. If so, less-compressed scores (= wider Scoring Spreads) would be less "lucky".

Third, columns 2 and 3 in the table on page 10 show that holes which assign scores by pure random luck are likely to contribute little (or negative amounts) to the Scoring Spread of Total Scores.

Fourth is the following experiment. I formulated a way to get holes to assign their scores to players based on varying degrees of luck.

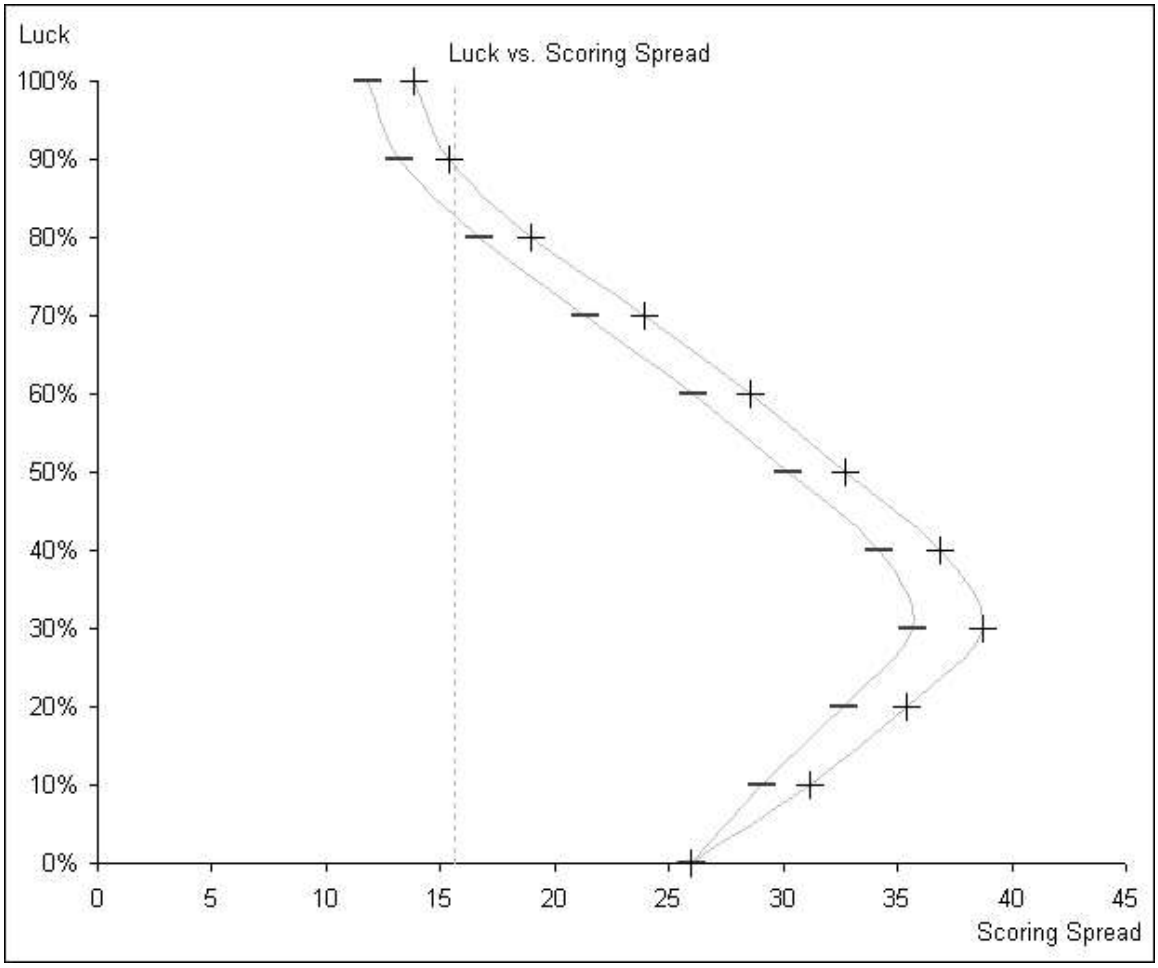
- At 0% luck, each hole gave the lowest score to the best player, the next lowest score to the next best player, etc.

- At 100% luck, each hole could assign any score to any player without regard to that player's skill.

- At 30% luck, the lowest score could go to any of the top 30% of players, the median score could go to any of the players in the middle 30% ($35^{th}$ %ile to $65^{th}$ %ile). and the worst score to the bottom 30% of players.

I ran several thousand rounds, added up the total scores from each hole, and computed the Scoring Spread of Total Scores.

Here are the results. 95% of the results landed in between the two lines.




What this shows is that at very low levels of luck, it would indeed be possible to widen Scoring Spreads by injecting a little bit of random results. This is because there are so few scores available to any hole, that to assign them in a regimented way creates long strings of tie scores. For example, at zero luck with the scores from this tournament, there would be 25 players tied for 2nd with a score of 39 (the best player would have been assigned both the only ace from hole #3 and the lone 2 on #9).

However, that low of a level of luck is nowhere near reality. Looking at where this tournament's Scoring Spread for Total Scores of 15.93 intersects the results, we can say that this course is somewhere around 85% luck, ±5% or so.

Thus, any widening of Scoring Spread is almost certain to be accompanied by (and to some extent caused by) a lowering of luck.